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Words, 'words', "words"
UICSM PROJECT STAFF, *University of Illinois*

The fifth year of teacher education for teachers of mathematics
DWAIN E. SMALL

Mathematics in French secondary schools
LAUREN G. WOODBY

College preparatory mathematics in grades nine through twelve
JACKSON B. ADKINS

The official journal of
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The Mathematics Teacher

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Words, 'words', "words", UICSM PROJECT STAFF, *University of Illinois* 194

The fifth year of teacher education for teachers of mathematics, DWAIN E. SMALL 199

Mathematics in French secondary schools, LAUREN G. WOODBY 204

College preparatory mathematics in grades nine through twelve, JACKSON B. ADKINS 209

The comparative effectiveness of one-level and three-level assignments in plane geometry, ERNEST R. GRIFF 214

Progress in mathematics instruction, JOHN R. MAYOR 217

DEPARTMENTS

Historically speaking,— PHILLIP S. JONES 220

Mathematics in the junior high school, LUCIEN B. KINNEY *and* DAN T. DAWSON 225

Memorabilia mathematica, WILLIAM L. SCHAAF 229

Reviews and evaluations, RICHARD D. CRUMLEY 232

Points and viewpoints, KENNETH E. BROWN 234

Testing time, ROBERT S. FOUCH *and* ROBERT KALIN 236

Tips for beginners, FRANCIS G. LANKFORD, JR., *and* JOSEPH N. PAYNE 241

Have you read? 198, 203, 243; *What's new?* 224

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

National Science Foundation Institutes for teachers of science and mathematics 244

Your professional dates 246

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Words, 'words', "words"

UICSM PROJECT STAFF,[1] *University of Illinois, Urbana, Illinois.*

Teachers of mathematics are learning that to say more precisely what one intends makes learning an easier task. The authors herein propose a means for building a more precise classroom and textbook language, and they show how it can be used.

Introduction

THERE ARE AT LEAST two ways to proceed in teaching someone the use of a new word. The first of these ways is that commonly used in teaching the meaning of a word to a very young child. You say the word and, at the same time, you point to or otherwise exhibit an object or an action to which the word applies. After several such performances, the learner tends to associate the word with the appropriate objects or actions. Finally, the learner reaches the stage where the word "calls to mind" the *referent of the word.*

A second way of teaching the meaning of a word is to make use of the learner's knowledge of other words. For example, all one needs to do to teach the meaning of the Latin word 'pons' is to tell the learner that it means the same thing as the familiar English word 'bridge'. Sometimes it takes a combination of familiar words to teach the meaning of a new word. Thus, 'confectioner' means the same thing as 'person who manufacturers or sells candy'.

Each of these two methods—the "pointing" method and the "new words for familiar words" method—must be used in teaching the meaning of the "words" in mathematics. It is the purpose of this article to examine some problems connected with the second method.

Names

To give a discussion on paper of the things around us in the world, you need *names* for the things.

When you use language to talk about things, you do not use the things themselves; if you talk about monkeys and giraffes, you do not need to have any monkeys and giraffes present, you need only the words 'monkeys' and 'giraffes'. Similarly, if you are talking about certain words, you do not need these *words* themselves; you need *names* for them. To give on paper a discussion about words themselves, you need a written name for each word you want to talk about. One might consider using a word as its own name, but frequently this will lead to confusion because the reader may not be able to tell whether the discussion concerns the words themselves or the *things* named by the words. Here is an example:

Mary has four letters.

Normally, you would take this sentence to mean that a girl has four pieces of mail or that she has won four "letters" in athletics. Suppose, however, that the writer intended that you understand by this sentence that the girl's name consists of four letters of the alphabet. The writer

[1] University of Illinois Committee on School Mathematics, Project for the Improvement of School Mathematics.

Max Beberman (Director), David A. Page (Editor), Herbert E. Vaughan (Mathematics Consultant), Gertrude Hendrix (Teacher Co-ordinator), *et al.* The ideas expressed in this article are basic to the texts, teacher commentaries, and teaching program of the project. This article and the other work of the project have been made possible by a grant from the Carnegie Corporation of New York.

needs to adopt some convention to indicate that he is talking about the name of the girl rather than the girl herself; he needs a *name* for the *name of the girl*. One method commonly used by logicians to form a name of a word is to enclose that word between single quotation marks. We shall use that convention throughout this article, and we propose that some such convention be used in more high school mathematics texts. According to this convention, each of the following sentences is a true sentence:

'Thomas Edison' is made up of two parts, 'Thomas' and 'Edison'; it contains twelve letters; and it doesn't contain very much ink, although it does contain more ink than does 'James Watt'. Thomas Edison was a man responsible for many important inventions.

Mary took four letters out of the mailbox, and she saw that each of them had 'Mary' written on it; so she knew that each was addressed to her. In this case, Mary had four letters, and on each envelope 'Mary' had four letters.

Numbers and numerals

If you think that in everyday language confusions of the "Mary has four letters" —type are infrequent, you are correct. However, confusions of this type often occur in the mathematics classroom and their elimination is the key to several teaching problems. Suppose you write on the blackboard the following:

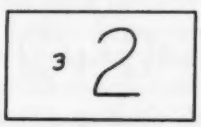

Figure 1

and ask your students, "Which is larger, two or three?" It is quite common to receive a reply such as, "Well, the two is bigger than the three, but three is *really* bigger than two." When the student says "the two" he means the symbol or name for 2; when he says that "three is really bigger than two" he is talking about the *numbers* two and three themselves. Thus, you can obtain agreement from the student that the marks are not the numbers themselves. If the student were familiar with the punctuation convention used in this article, he would say: the '2' is larger than the '3', but 2 is smaller than 3.

If the number symbols on paper are not numbers, what are they? They are *numerals*. The word 'numeral' is commonly used in a variety of ways. Sometimes it is used to mean a single Arabic digit. We shall use 'numeral' as a synonym for 'number name'. Thus '5', '153,718.92', 'XVI', '$\frac{3}{4}$', '$2+5$', and '32.7×-4.56' are all numerals. For example, we even call 'twenty-three' and 'eight divided by three' numerals.

If you tell students that the marks which they make for numbers are numerals and not numbers, a natural reply is "What are numbers then?" The student writes 'Empire State Building' and can point to the Empire State Building, whereas he writes '5' but cannot point to 5. One of the immediate advantages of pointing out the distinction between numbers and numerals is that it raises the question: What are numbers? In a conventional course, students can be told that the number 5 exists independently of its name just as, for example, does the concept *justice*. The student never thinks that the word 'justice' is justice itself and in a similar way he can appreciate that the numeral '5' is not 5 itself. He has more or less vague ways of deciding whether a given act exemplifies justice (i.e., is just). He is likely to be much surer in deciding whether a given set exemplifies fiveness (i.e., has five members).

A number has many names

Students should learn that numbers have many names. For example, each of the following is a name of 3:

three $\quad 2+1 \quad \frac{6}{2} \quad$ III

$2\times1.5 \quad 300\% \quad (9000-8997)$

Of course, this list can be extended indefinitely. An interesting elementary exercise which students enjoy is based on the

idea that a number has many names. List on the blackboard (or, preferably, duplicate such a list) many number names, including different names for each of several numbers. The student's job is to classify this jumble of numerals by making a new list with the names of each number collected in a separate bunch. For example, the student might be given:

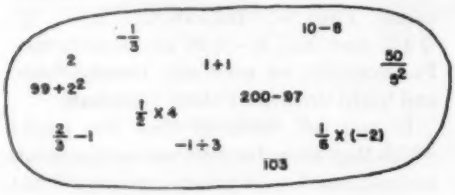

Figure 2

and he would organize the names as:

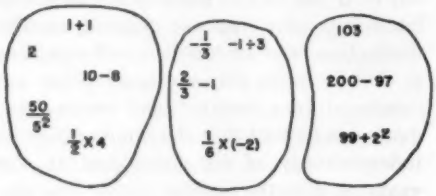

Figure 3

This type of classification exercise provides an interesting drill in such areas as simplification of radicals and work with decimal and common fraction equivalents.

Simplification of Names

One of the results of confusing a number with its name can be observed when students are introduced to the topic of binomial surds. If you write:

$$3+\sqrt{2}$$

on the board and ask such questions as:

Is this one number or two numbers?
Are these two numbers added?
Can you add these two numbers?

you will get a variety of answers. Some students will say that it is a case of "indicated addition," but if you ask for the distinction between indicated addition and "ordinary" addition, you will not get a clear answer. Other students will tell you that you cannot add 3 and $\sqrt{2}$! Yet, many of these students at some other time will agree that given *any* two numbers, there is a third number which is their sum. The difficulty these students are having is confusing names of numbers with the numbers themselves. In grade school they learned to view a symbol such as

$$3+6$$

as a command to do something: get rid of the plus sign. Thus, they didn't believe that they had added if there was still a plus sign present. They didn't realize that '3+6' is itself one of the many names for that number which is the sum of 3 and 6. In going from '3+6' to '9', one has found a *simpler looking name* for this number.

Now *some* numbers do have names made up of digits alone. Other numbers do not. The number $3+\sqrt{2}$ is an example of a number which does not have such a "standard" name. Neither has it been given any name simpler than '$3+\sqrt{2}$'. But $3+\sqrt{2}$ is *one* number! This number has many names. Among its many names are:

$$1+\sqrt{2}+2 \qquad 2^{1/2}+3 \qquad 4+\sqrt[4]{4}-1$$

$$\frac{7}{3-\sqrt{2}} \qquad 5\left(\frac{3}{5}+\sqrt{\frac{2}{25}}\right) \qquad 3+\sqrt{2}$$

Most people would agree that '$3+\sqrt{2}$' is the simplest looking of these names. If someone were using this number repeatedly, however, he might *give* it a still simpler name. 'π' and 'e' are examples of simple names which people have given to certain numbers which are frequently used. (One should not lose sight of the fact that the simplest name of a number is not always the most useful name. For example, to find a simple name for the sum of

$$\frac{4}{7} \text{ and } \frac{8}{17},$$

it is convenient to use the less simple names

$$\frac{\text{'68'}}{119} \quad \text{and} \quad \frac{\text{'56'}}{119}$$

of the addends.)

Complex numbers

Understanding the distinction between a number and its names is even more important when the student works with complex numbers. When students are asked about adding 3 and $\sqrt{2}$ some of them will suggest that you could do it by using 1.414 instead of $\sqrt{2}$. Of course, this is changing the problem into a new one because '1.414' is not a name of $\sqrt{2}$. Some students may not realize that the problem is changed, and other students with a more sophisticated understanding may be comforted by the fact that $\sqrt{2}-1.414$ is a very small positive number so that the desired sum has "almost been obtained."

But what about $3+4i$? Here the plus sign cannot be removed even by approximation unless a new simple name is arbitrarily coined. Once more the student should learn that

$$\text{'}3+4i\text{'}, \quad \text{'}4i+3\text{'}, \quad \text{'}1+4i+2\text{'},$$
$$\text{'}i+3+3i\text{'}, \quad \text{'}2(1\tfrac{1}{2}+2i)\text{'}, \cdots$$

(again, the list might be continued indefinitely) are all names for one complex number. There is no number which is close to the given number and which has a simpler name. You can't avoid all '+' and '−' signs in names for complex numbers (neither of whose components is 0) unless you use an entirely different naming system (for example, exponential notation).

The difficulty with complex numbers mentioned in the last paragraph is only one of the problems confronting the student when he first learns about these numbers. The student has invariably been told that the square of every number is nonnegative. Then suddenly, at the whim of the text and the teacher, the old rule is forgotten and the student is supposed to accept the new number i and its relatives. It is moderately easy to fool students who believe that numbers are marks on paper because they think they *see* the number i. But as soon as students learn that numbers have a separate existence from the marks which name them, they will ask how you know there can be a number whose square is negative. Such students would be justified in making the counter-proposal that a new number h be created such that when it is added to any number the sum is 10. This creation of h is analogous to the usual classroom introduction of i. Clearly, assuming the existence of such a number h plays havoc with ordinary arithmetic. But what do you do to convince students that the new number i can be introduced without damage? The answer to this question comprises a projected article, but suffice it to say here that you do not merely introduce a new number i; rather using the real numbers, two at a time, as building blocks you build an entirely new number system of which i is one number. Carathéodory sums up eloquently the historical problem and the problem of the student who is first confronted by complex numbers:

> Now just as one can derive the totality of real numbers from that of the positive real numbers by adjoining to the latter the number -1, so one can also try to regard the symbol $\sqrt{-1}$ as a new admissible number which must be adjoined to the real numbers. The mathematicians of the seventeenth and eighteenth centuries did this quite naively, and they were led to the most astonishing results in their calculations, so that they were inspired to follow up the method further and to apply it to one new problem after another. However, they never realized at all clearly that the new method of calculation could be rendered free of contradictions, and for this reason they called $\sqrt{-1}$ "the imaginary number."[2]

Now that we have defined the complex numbers, one and all, as the elements of a field that contains the real numbers in the complex domain and the number i as well, nothing remains of the difficulties which gave the mathematicians of the eighteenth century such headaches. These difficulties were all due to the fact that

[2] Carathéodory, *Theory of Functions of a Complex Variable* (New York: Chelsea, 1954), p. 2.

one wanted, at that time, to adjoin to the ordinary real numbers a number i which would not fit in anywhere.[2]

More implications

We have indicated some of the reasons why the distinction between a name and its referent is important. (We have saved until another article the changes implied in the teaching of plane geometry.) An even stronger reason is that an understanding of this idea is essential to a satisfactory, contradiction-free understanding of the role played by letters of the alphabet in algebra. Many of the difficulties involved in teaching the use of *variables* can be eliminated after securing such a foundation as has been indicated in this article. We plan to treat variables in a future article.

[2] *Ibid.*, p. 7.

Have you read?

Cartwright, M. L. "An Enquiry Into the Teaching of Mathematics," *Mathematical Gazette*, October 1956, pp. 199–200.

This is an interesting report from a commission set up by the International Mathematics Union. The commission is attempting to determine mathematics content on the high school level in many countries. This is only an interim report. A few of the results are:

1. Deductive geometry is important, but the order is no longer Euclidean.
2. Solid geometry is limited to informal work.
3. Trigonometry is only elementary and mostly triangular measure.
4. Analytic geometry is seldom found in high school.
5. Spherical trigonometry is rarely given.
6. Philosophy and History of Mathematics are touched on certain occasions.
7. Mechanics is a part of mathematics rather than physics.

I think you will be interested in this report and in comparing it with your offerings.

Court, Nathan Allshiller. "The Might and Plight of Reasoning," *Scripta Mathematica*, March 1956, pp. 45–52.

Many times you have stopped to reflect on the art of reasoning. But have you ever asked yourself whether it was a logical or psychological activity? Or is it a physical activity? When you listen to music, are you using your whole physical being? Just what is empathy?

For that matter, what is reasoning? The author proposes that it is a physical operation which occurs in the mind. It is experimentation in the abstract. He gives some very good illustrations and points out how geometry plays an important role in this activity. As an example, what happens in proof of congruency and the idea of superposition?

Can we improve reasoning? Can we teach reasoning? If it is physical activity in the abstract, the answer would seem to be yes. But the author points out two serious problems: keeping aware of all the links in a reasoning chain, and recognizing a link when we see it.

Here is the $64 question: What causes one to choose his particular ideas?

Haskew, L. D., "Advanced Study for Exceptionally Able High School Students," *Exceptional Children*, November 1956, pp. 50 and 89.

This is a report of an interesting project carried on by the University of Texas during the summer of 1956. There were twenty-eight selected high school students from fourteen schools who participated in the program. They spent all day, five days a week, in the field of chemistry; hearing lectures, working in laboratories, seeing and doing demonstrations, and developing projects. You will be interested in the statements concerned with the evaluation of the program. Such statements as "high school students are smart and sound thinkers" are pleasing to hear.

The program also gives opportunity for early orientation, information about college instruction, and plans for the future. You will be pleased to read about the faculty reactions to the program. You should carry this information to your state university or local college.—Philip Peak, *Indiana University, Bloomington, Indiana.*

The fifth year of teacher education for teachers of mathematics

DWAIN E. SMALL, *Southern Illinois University, Carbondale, Illinois.*

This study indicates teacher preference as to the type of work that should constitute a fifth year of study. It is notable that teachers still give a high rank to courses in mathematics as courses which fulfill their needs.

THE INCREASED DEMAND for graduate study by teachers whose experience and educational backgrounds are extremely varied presents a serious problem to educational institutions that are offering or preparing to offer a graduate program for teachers. The increased demand for graduate study is a result of many factors. Some school systems are encouraging teachers to obtain advanced work through contract and salary agreements. Other systems are requiring a master's degree before tenure can be obtained. Some systems are awarding salary increases, promotions, and assignments on the basis of further education.

Professional organizations have contributed to the problem by encouraging teachers to improve themselves so that our youth might have better guidance in their efforts to become good citizens in our democratic society. This is evidenced in the increased number of symposiums and conferences sponsored by professional organizations and educational institutions on the fifth year of teacher education.

During the summer of 1953 a symposium was held at the University of Wisconsin sponsored by the National Council of Teachers of Mathematics and the Mathematical Association of America. This symposium concerned itself with the present programs in mathematics and indicated the need for a study of the graduate mathematics programs. In the light of this felt need, the preliminary preparation for this investigation was begun in 1953.[1]

A fifth year of teacher education is distinguishable from a five-year internship program, because the philosophy and curriculum organization of the two plans are quite different. The fifth-year program assumes that the teacher has completed a bachelor's degree with the professional education courses and subject matter courses necessary for teacher certification. The fifth year intends to prepare a master teacher through advanced study in professional education, subject matter, and related fields while the teacher is participating full-time in teaching activities. The internship program is a part-time teaching and study program.

Since the undergraduate backgrounds of teachers entering the fifth-year program are quite different, the program of the fifth year cannot be a fixed program. Those teachers who have a strong subject matter background should have a program to strengthen their professional and cultural background. Those teachers who have a weak subject matter background must have a strong subject matter program in the fifth year.

[1] Dwain E. Small, "Opinions of Secondary Mathematics Teachers Concerning the Fifth Year of Teacher Education" (Doctoral dissertation, Indiana University, 1955), p. 228.

Why are teachers, many of whom are weak in subject matter preparation, refusing to take courses in their subject matter areas during the fifth year? This question cannot be seriously answered by stating "the teachers are taking the easy way out." This probably is true for some teachers. Others are anticipating a future opportunity for an administrative assignment. What about the teacher who is to be a permanent mathematics teacher? In the past, the needs of this teacher have not been met due to the lack of co-operation between the departments of education and mathematics, and to the neglect in considering the opinions of teachers concerning the fifth-year program.

If the teaching of mathematics in our secondary schools is to be improved, the fifth year of teacher education for teachers of mathematics must be improved. Many groups should have a part in the development of the program of the fifth year. The secondary mathematics teachers are one of these groups. In addition, representatives from professional mathematics, mathematics education, industry, and business should share in the development of the program. In this study a survey was made of the opinions of a group of secondary mathematics teachers concerning the fifth year of teacher education for teachers of mathematics. It is hoped that educators and mathematicians will not ignore the opinions of these secondary mathematics teachers when revisions of the curriculum of the fifth year are undertaken.

Through the co-operation of the National Council of Teachers of Mathematics the mailing list of the Council was used to obtain a sample of 1465 teachers who represented every state in the United States. The mailing list was stratified by regional accrediting associations, and the sample obtained by the use of random numbers. A questionnaire of 52 items was sent each of the 1465 names selected from the membership roll. There were 951 returned questionnaires, of which 878 were from secondary mathematics teachers. Of this group of teachers, there were 132 women in junior high schools, 84 men in junior high schools, 346 women in high schools, and 316 men in high schools.

To give here a complete analysis of the sample character would involve too much detail. Let it suffice to say that each control factor in the study contained a well-distributed representation. It is interesting to note that 67.1 per cent of the sample had more than ten years of teaching experience. As to the mathematical background of the sample, 18.1 per cent had less than 20 semester hours of undergraduate mathematics, and 27.7 per cent had taken no graduate mathematics.

Each of the participating teachers was asked to respond to each statement in the questionnaire. The response could be complete agreement, tend toward agrreement, tend toward disagreement, or complete disagreement. In addition to the opinion response, a space was provided for teachers to check whether they had taken the specific course on the undergraduate or graduate level.

In the analysis, a response of complete agreement was given a weight of 1.00; tend toward agreement, a weight of 2.00; tend toward disagreement, a weight of 3.00; complete disagreement, a weight of 4.00. These weighted responses were averaged to obtain a weighted response index for each statement. The index was then used to give each statement a rank order.

It was necessary for interpretation purposes that an arbitrary cutoff point be determined for significant agreement and significant disagreement. This cutoff point could vary depending on the use to be made of the analysis. For this study, any weighted response index below 2.00 was considered significant agreement and a weighted response index above 3.00 was considered significant disagreement.

There were 27 statements with which teachers indicated significant agreement. These statements are given below in rank order with the index following each one.

The fifth year of teacher education for

teachers of mathematics should:

1. Provide an advanced course in the methods and problems of teaching mathematics in the secondary school. (1.33)
2. Provide adequate laboratory facilities so that teachers may have experience working with audio-visual, curriculum, and laboratory materials related to mathematics. (1.35)
3. Have an instructional staff which has had recent experience with secondary pupils. (1.39)
4. Provide all teachers with an opportunity to obtain knowledge and skill in the use of remedial activities in teaching mathematics. (140)
5. Use instructional methods which emphasize demonstration, class participation, and the use of audio-visual materials. (1.41)
6. Provide workshops for the development of the skills needed in the supervision and direction of the mathematics laboratory as an integral part of the mathematics classroom. (1.49)
7. Provide a course in the history of mathematics. (1.51)
8. Provide a course in basic number theory. (1.55)
9. Provide a course in the methods of teaching arithmetic in the secondary schools. (1.58)
10. Provide workshops that combine the areas of mathematics and science for a better understanding of their interdependence. (1.59)
11. Use campus laboratory schools or co-operating schools for demonstration, research, and experimentation. (1.59)
12. Provide a course in educational measurements which will give the teacher a knowledge of the basic principles involved in informal achievement-test construction and interpretation. (1.60)
13. Provide a course in mathematical statistics. (1.60)
14. Provide a course in modern algebra. (1.65)
15. Encourage at least one year of full-time teaching experience before entering the program. (1.69)
16. Provide the teacher with an opportunity to acquire an understanding of the principles of individual and group guidance. (1.70)
17. Provide a course in modern geometry. (1.71)
18. Provide teachers with an opportunity to improve their cultural background by taking some work in cultural fields outside of the fields of mathematics and professional education. (1.72)
19. Provide a course in the theory of equations. (1.73)
20. Provide opportunities for the teacher to apply and extend his professional competence through directed experience with young people. (1.74)
21. Provide opportunities for the study of adolescent growth and development using the techniques of demonstration, observation, and direct contact. (1.77)
22. Require some courses in the physical sciences. (1.77)
23. Provide a course in the mathematics of finance. (1.80)
24. Use some visiting professors for instruction in the summer session. (1.83)
25. Require not more than 25 per cent of the work to be done in professional education. (1.91)
26. Be a flexible program with few specific course requirements in either mathematics or education so that a teacher may select those courses which best meet his needs. (1.91)
27. Provide in-service courses for which credit is received for individual or group study of local school problems. (1.92)

There were only two statements with which the teachers were in complete disagreement. These statements were inserted in the questionnaire as internal checks of consistency. It should be noted that they are contradictions of the state-

ments ranked 5th and 25th above. The statements are the following:

The fifth year of teacher education for teachers of mathematics should:

1. Use the lecture extensively as the method of instruction. (3.23)
2. Require at least 50 per cent of the work to be done in professional courses in education. (3.24)

In response to the question, "What approximate per cent of the total program should be devoted to each area?" the teachers indicated the following percentages:

Professional education courses	22.4%
Mathematics courses including laboratory	46.8%
Research	10.0%
Observation and advanced student teaching	15.0%
Other (Physical science, cultural, and travel)	5.8%

According to the secondary mathematics teachers, approximately one half of the fifth year should be devoted to mathematics. This should include laboratory and workshop courses. The distribution of the program would depend upon the previous educational background of the teacher entering the program of the fifth year.

The chi-square test was used to determine the dependency of the responses on whether the teachers had taken a specific course, and it was found that the opinions of the responding teachers were dependent on whether they had done so. On all statements except one, those teachers who had taken a specific course were more inclined to agree with the statement than those who had not taken the course. On the statement concerning the provision of a course in modern algebra, those who had taken the course were less inclined to agree This could be attributed to the presentation of the course, instructor, and many other factors, but it is unlikely that all of the 253 teachers who had taken a course in modern algebra had poor instructional conditions. Since those who had not taken a course in modern algebra were more inclined to agree, it is probable that they have been deceived by the title "modern." In addition, the values claimed for modern algebra in our professional journals have possibly not been fully realized by those who have taken the course.

The teachers were asked if a thesis should be required for the completion of a master's degree. Those who had completed a thesis were in complete agreement and those who had not completed a thesis were in complete disagreement. This could mean that experience with the thesis is a determinant of its value, or it might be interpreted as an example of wanting to maintain standards with which the individual has complied.

This study would indicate that an opinion concerning the value of a course should be based on experience with the specific course. It is possible that a felt need for a course can be expressed by those who have not taken the course if the true content and value of the course have been sufficiently described in our professional journals and meetings. It is imperative that teachers keep themselves alert to their needs through professional reading and attendance at professional meetings.

The major conclusions drawn from the data of this study can be summarized as follows:

1. The opinions of a teacher concerning a specific course are dependent on whether the teacher has taken the course.
2. The fifth year of teacher education for teachers of mathematics should include work in professional education, research, mathematics, advanced student teaching, and cultural areas. Approximately 50 per cent of the work should be done in mathematics courses, including laboratory, and not more than 25 per cent of the work should be done in professional education courses.
3. The program of the fifth year should be broad and flexible with few specific

course requirements in either mathematics or education.

4. The teachers expressed a need for a better understanding of the methods and problems of teaching mathematics and for knowledge and skill in the use of remedial activities in the secondary school. The teachers also expressed a need for a course in the teaching of arithmetic on the secondary level.
5. Adequate audio-visual, curriculum, and laboratory facilities related to mathematics should be provided.
6. Workshops which relate the areas of mathematics and science, as well as develop the techniques for the laboratory as an integral part of the mathematics classroom, should be provided.
7. Courses in the history of mathematics, basic number theory, mathematical statistics, modern algebra, modern geometry, theory of equations, and mathematics of finance should be provided in the fifth year.
8. Courses in educational measurements, guidance, in-service courses concerning local school problems, and adolescent growth and development should be provided in the fifth year.
9. The fifth-year program should have an instructional staff which has had recent experience with secondary pupils, and should use some visiting professors for instruction in the summer session.
10. Some work should be required in the physical sciences, and an opportunity be offered to teachers to extend their cultural backgrounds.
11. At least one year of full-time teaching experience should be encouraged before a teacher enters the program of the fifth year.
12. Campus laboratory schools or cooperating schools should be used for demonstration, research, and experimentation, so that opportunities may be provided for teachers to apply and extend their professional competence through directed experience with young people.
13. Instructional methods should be used which emphasize demonstration, class participation, and the use of audio-visual materials.

It has been recommended that curriculum revision committees use the conclusions of this study to form the basis for experimentation in their own institutions. The investigator feels that the problem of the fifth year is not a local or state problem, but a national problem. It is recommended that state study groups be organized to work on the problem of the fifth year program. These study groups could originate within the State Councils which are affiliated with the National Council of Teachers of Mathematics. Co-ordination and integration of findings would undoubtedly improve the program of the fifth year for teachers of mathematics.

Have you read?

MAISEL, ALBERT Q. "'Doc' Harrington's 'Dawn Patrol of Young Scientists'," *Reader's Digest*, November 1956, pp. 142–146.

Here is an account of what happens to students when they are given a little encouragement and a lot of opportunity. I admit 4:30 a.m. seems rather early for any teacher to start inspiring his students. This is what "Doc" Harrington often did. You will also be interested to know that this started in Albuquerque back in the depression years when most schools were cutting back offerings for scientists. The mathematics, shop, and science teachers of Albuquerque refused to join the parade and since that time have built a feeling for the sciences in the school that is probably unequalled. They carry this interest out into the grade schools, and it gains momentum throughout the boys' and girls' schooling. You will like this article, whose philosophy is stated in the last two lines: "There isn't a single thing we've done here that other schools couldn't do as well. The real trick is to get down to doing it."—PHILIP PEAK, *Indiana University, Bloomington, Indiana.*

Mathematics in French secondary schools

LAUREN G. WOODBY, *Central Michigan College, Mt. Pleasant, Michigan.*

In these days of criticism of secondary mathematics, it is good to have comparative studies of educational programs. This article describes the program in France and touches on current educational problems.

INITIAL IMPETUS for this report came from discussions within the Committee on International Relations. The purpose of the report is to furnish mathematics teachers in this country with some current information about the teaching of mathematics in the secondary schools of France.

An examination of mathematics education in France requires at least a brief study of the French philosophy of education and its organization to provide a suitable frame of reference.

For more than a century, secondary education in France has been characterized by the expression *culture generale* and the ultimate goal has been the development of a cultured elite. Two dominant factors in the French educational theory are respect for tradition, reflecting back to the scholasticism of the Middle Ages, and the principle of centralization firmly established by Napoleon.[1]

Although we find much discussion of "sweeping educational reforms" both in the literature of comparative education and in the French periodicals, these changes are occurring slowly, and they have been piecemeal. The Langevin-Wallon Commission in 1947 produced a complete plan for overhauling the national educational system. Among the proposed reforms were adoption of a child-centered curriculum, abolition of most of the examinations, raising of the compulsory school age from 14 to 16, improvement of the training of teachers, and expansion of the opportunities for the youth of secondary school age. The report of the Langevin Commission[2] has never been enacted into law. However, the report has been generally accepted and has inspired many postwar reforms. It forms the basis of the educational reform bill of the Ministry of Education proposed to the National Assembly in August 1955.[3]

Secondary education in France extends from age 11 through 17 or 18. The first (orientation) cycle consists of four years of classes called sixth, fifth, fourth, and third. After completing this first cycle, many pupils terminate their education. They may obtain the *brevet d'etudes du premier cycle du second degré* (B.E.P.C.) by passing the examination. The second (determination) cycle consists of three years of specialization in classes called the second, first, and terminal. The first part of the formidable examination for the *Baccalauréate* is taken upon completion of

[1] Donald W. Miles, *Recent Reforms in French Secondary Education* (Bureau of Publications Teachers College, Columbia University, 1953).

[2] Letter to author from Pierre Donzelet, General Director to the Ministry of National Education, Permanent Representative of French Universities in the U. S., 972 Fifth Avenue, New York, dated January 9, 1956.

[3] *Project de Loi Portant Réforme de l'Enseignement Public. No.* 11404. Assemblée Nationale Deuxieme Législature. Session de 1955.

six years. Those who are successful pass on to the terminal class and specialize further. The second part of the *Baccalauréate* is taken at the end of this seventh year.

The current program of studies

It may be of interest to mathematics teachers in this country to know what mathematics is studied in the various classes. The program in mathematics, as in other subject areas, is uniform and is spelled out in careful detail by decrees and circulars. The following abbreviated topic summary indicates the content of the First Cycle.[4]

Sixth class (age 11):

Operations with whole numbers, decimals. Metric system. Rule of three. Measure of length: use of instruments. Measure of area: rectangle, right triangle, circle, surfaces of simple solids. Measure of volume: cube, rectangular parallelepiped, right prism, pyramid, cone. Measure of weight. Measure of angles. Measure of time. Uniform motion. Percentage and simple interest.

Fifth class (age 12):

Arithmetic—Decimal and common fractions. Remainders in division by 2, 3, 5, and 9, proof by 9's. Literal representation. Special products and factoring. Equation solving, checking.

Geometry—Line segments, plane figures, angles. Three cases of congruent triangles. Construction: use of ruler, compass, T square, and double-decimeter.

Fourth class (age 13):

Arithmetic—Practice with fractions. Decomposition of a number into prime factors. Finding G.C.D. and L.C.M. of a group of numbers.

Algebra—Directed numbers: operations, inequalities. Representation of a point on an axis. Elements of algebraic calculation. Powers: multiplication and division involving exponents, zero and negative exponents. Monomials. Polynomials of one variable. Equations of first degree in one unknown.

Geometry—Triangles. Parallel lines. Polygons and their properties. Circles: arcs, chords, tangents, intersections of lines related to circle, measure of angles. Constructions.

[4] *Nouveaux Horaires et Programmes de l'Enseignement du Second Degré*. Librairie Vuibert, 63 Boulevard St. Germain, Paris. 18th edition.

Third class (age 14):

Arithmetic—Ratio and proportion. Square root: use of tables, rule (without proof).

Algebra—Polynomials of one variable. Rectangular co-ordinate: graphs of linear functions. Simultaneous linear equations in two unknowns.

Geometry—Necessary and sufficient conditions, converses. Proportional division of a line segment. Similar polygons. Regular polygons and the circle. Trigonometric relations of an acute angle. Areas: triangles, quadrilaterals, sectors.

Additional topics for preparation for the B.E.P.C. include quadratic equations, homothetic figures, and geometry of space, with applications to the terrestrial sphere.

The mathematics content of the second cycle depends upon which of the following eight sections a student chooses:

A Classical
A′ Classical scientific
B Modern
C Modern (with Latin and two modern languages)
C′ Modern (with Latin and one modern language)
M Mathematics (with two modern languages)
M′ Mathematics (with one modern language and natural science)
B Technical (with two modern languages and economics)

For the sake of brevity, the topical outline of the Second Cycle which follows is for only one section, C:

Second class (age 15):

Algebra—Functions of one variable, increments. First and second degree equations. Roots: existence, sign, sum, product. Vectors.

Geometry—Similar triangles, Thales' theorem, Pythagorean relation. Trigonometric relations. Symmetry. Harmonic division. Cosine law. Radian measure.

First class (age 16):

Algebra—General equation of second degree in one unknown. Definition and geometric significance of the derivative of a function. Variable rectilinear motion.

Geometry—Lines and planes in space. Orthogonal projection. Skew lines. Intersections. Geometrical addition of vectors.

Trigonometry—Circular functions: periodicity, relations between the functions. Trigonometric identities and equations.

In the terminal year that follows, further specialization is possible. There are some options: philosophy-letters; experimental science; mathematics; and, since 1949, technical and economic science. The academic load for the terminal year appears rather formidable—32½ class hours per week. In the mathematics option, nine hours per week are devoted to mathematics and cosmography. Some of the topics included in the mathematics class follow:

Arithmetic—Number system: prime numbers, factors, theorems of operations with natural numbers. Approximate numbers.

Algebra—Function concept. Increments. Continuity. Differentiation and integration. Progressions and compound interest.

Trigonometry—Solution of the general triangle using logarithms. Sine law and cosine law. Differentiation of trigonometric functions.

Kinematics—Rectilinear and curvilinear motion. Vectors. Simple harmonic motion. Trajectories

Statics—Composition and resolution of forces. Mass. Equilibrium.

Geometry—Additional work emphasizing applications. Translation and rotation in a plane and in space. Poles and polars. Inversion. Conic sections.

Descriptive Geometry—Intersections. Determination of distances and angles by projection.

Cosmography—The sky and the constellations: celestial sphere, co-ordinate systems, diurnal movements. The earth: geographic coordinates. The solar system: solar spectra, apparent movements, ecliptic, seasons, system of Copernicus, Laws of Kepler, Newton's Law. Measurement of time: calendars, universal time, siderial time, civil time. Projections: Lambert's projection, Mercator's projection.

The emphasis placed on examinations was made clear to the writer by reading an issue of *l'Éducation Mathématique*.[5] This bimonthly journal consists solely of questions from the various nationwide examinations in mathematics. It is published especially for the pupils who are not yet able to follow the *Journal de Mathématiques Élémentaires*,[6] also a collection of examination questions. These journals are evidently widely used, both in and out of class, as a means of preparing for the examinations. Both have been published for more than fifty years. Solutions to the problems are invited and are published within two or three months. Credit is acknowledged those who send in solutions in the three categories—very good, good, and passable. The following samples extracted from these two journals perhaps indicate the goals of mathematical instruction more clearly to an American reader than does the program of studies.

A. (From B.E.P.C. examination given in Indochina, June, 1955.)

1st. Factor completely

$(12x^2-3)(x+3)+(x^2-9)(2x-1)$.

2nd. Simplify

$$\frac{(12x^2-3)(x+3)+(x^2-9)(2x-1)}{4x^3-x}$$

and show that, except for certain indicated values of x, the expression reduces to

$$y=\frac{7x+21}{2x+1}.$$

3rd. a. Determine x so that y takes the value 7/2

b. Determine x so that y takes the value

$$\frac{7+\sqrt{3}}{2}.$$

Calculate x to the nearest hundredth.

4th. For what values of x are the two terms of the algebraic fraction simultaneously positive?

The above is one problem. (Note that the solution to 3(a) is: No finite values of x.)

B. (From Second Class, June, 1955.)

Construct an isosceles triangle ABC given the vertex angle A and the sum of the altitude and the base, $AH+BC$.

[5] L'ÉDUCATION MATHÉMATIQUE. Librairie Vuibert. 63 Boulevard St. Germain, Paris.

[6] JOURNAL DE MATHÉMATIQUES ÉLÉMENTAIRES. Librarie Vuibert. 63 Boulevard St. Germain, Paris.

C. (From the competitive examination for admission to the School of Engineering, Marseilles.)

Draw, on the same rectangular axes OX and OY, the curves representing the following functions.

$$y=\tan x+\cot x$$

$$y=\sin x+\cos x$$

and show that the equation $\tan x+\cot x=\sin x+\cos x$ has no solution.

D. (From Baccalaureate first part, Metropolitan, France, June 1955.)

1st. Draw the curve C representing the function

$$y=\frac{-x^2}{4}+3x-6.$$

Show the axis of symmetry, D. Determine the intersection of C with D and with the co-ordinate axes OX and OY.

2nd. Construct on the same graph the Curve C' representing the function

$$y=\frac{x^2}{4}-x.$$

Determine the co-ordinates of A and B, the points common to the two curves C and C'. Use the graph to solve the inequality.

$$\frac{-x^2}{4}+3x-6>\frac{x^2}{4}-x.$$

Check the results by calculation.

3rd. A point M whose ordinate is m is on the Y axis. Express the function MA^2+MB^2 as a function of m. Determine m such that $MA^2+MB^2=50$. Discuss according to the value of the parameter k, the number of points M such that $MA^2+MB^2=k$.

An item of interest concerning the notation used in the last problem above is found in a short article[7] by Pierre Dedron, Inspector General of Public Instruction.

The square of a line segment AB is to be denoted as AB^2 rather than $(AB)^2$ or $\overline{AB}^2$. The notation $\overline{AB}$ is reserved for the sole purpose of indicating an algebraic measure and the notation AB is to indicate its magnitude.

We in this country can ascertain the viewpoint of our colleagues in France by reading their current professional periodicals. For example, in the lead article in the December 1955 publication of the Association of Professors of Mathematics, President Walusinski discusses "Mathematics and the Reform of Teaching." He cites two basic facts:

1. The continued increase of numbers of students at the secondary level.

2. The increasing importance of mathematics in the twentieth century.

He then poses the following questions based on the above facts:

1. Does the increase of numbers affect the quality of our teaching?

2. Does not the evolution of the sciences and mathematics force us to change our methods and our programs?

In his discussion of the questions, he points out that the dogmatic method is obsolete; students learn in diverse ways and through various media, but that this takes more time. He believes that methods need to be changed.

Walusinski concludes his article with four reform objectives of the Committee of the Association. (This committee is similar to our Board of Directors of the National Council of Teachers of Mathematics.) These objectives are:

1. *General principle:* the teaching of secondary mathematics cannot be done effectively unless (a) no more than 30 students are in a class and (b) at least four hours per week are scheduled.

[7] BULLETIN DE L'ASSOCIATION DES PROFESSEURS DE MATHÉMATIQUES DE L'ENSEIGNEMENT PUBLIC. 29 rue d'Ulm, Paris (March 1955).

2. *Directed study:* for all classes in mathematics, one hour per week in classes of half size.

3. *Secondary and vocational education:* better co-ordination of the programs to facilitate shifting from one course to the other.

4. *Programs of study:* to add suitable ideas of modern mathematics.

The same issue contains a report of a conference with the director general of technical training. He announced the innovation of an additional type of baccalaureate for girls called the *Baccalauréate Technique Economique*. The significance of this action is twofold, increased educational opportunity for girls in the technical fields, and the addition of another less academic road to the baccalaureate.

The University Committee of Teaching Information devoted the November 1955 issue of their monthly publication[8] to the teaching of mathematics. The three most prominent topics that were discussed were modern mathematics, statistics, and the co-ordination of physics and mathematics. Evidently the problem of modern mathematics in the secondary school has evoked much more interest and discussion in France than it has in this country. Among the questions discussed were: Is it necessary to oppose classical mathematics and modern mathematics? Does the mode of writing modern mathematics have creative virtues? If these new ideas are introduced into secondary education, how can the teachers become competent? (One reply:—This is the only problem. There are two kinds of teachers, a minority, young [under 60 years], curious, who keep up to date. . . . The others set their course once and for all and refuse to change it.) Is a reform of this kind compatible with the fact that secondary schooling is taken by an increasing number of students?

Professor Servais, president of the Belgian Society of Teachers of Mathematics, gives some examples of logic and modern notation which he believes to be so fundamental that they belong in the elementary mathematics curriculum. Notions included are propositions, sets, necessary conditions, sufficient conditions, the notation $P \rightarrow Q$, equivalence, conjunction, and intersection. This article was a reprint from the Belgian periodical *Mathématica et Paedagogia*.

"The Mathematics That Should Be Taught" is described in a letter from Professor Gattegno, of the University of London, to Professor Walusinski. Gattegno recommends that the teaching of mathematics and physics should be coordinated. His proposals include: start in the sixth class with vector representation; use demonstrations to show the effect of vectors; use concrete means to introduce the ideas of addition, multiplication by a scalar, and a negative number as equivalent to a rotation of 180°. These lessons in mathematical experience are not to be rigorous but should be a codification of experience. He states that characteristics of a vacuum tube are not too difficult to study at the age of 12. This topic brings in the use of squared paper and the drawing of a graphic relation.

One cannot escape the feeling that mathematics education in the secondary schools of France is highly intellectualistic. One major problem, and the same problem exists in our country, is that of providing suitable instruction in mathematics for the increasing numbers of youth who take vocational and semiprofessional courses.

For those who are interested in adding the *Bulletin de l'Association des Professeurs de Mathématique* to a library, the journal is published five times per year, October 15, December 1, January 15, March 1, and May 15, plus three special examination issues, November, the Baccalaureate, and the B.E.P.C. The cost is 1000 F. per year, approximately $3.00.

[8] "L'Enseignement des Mathématiques." CAHIERS PÉDAGOGIQUE POUR L'ENSEIGNEMENT DU SECOND DEGRÉ. Service d'Edition et de vente des Publications de l'Education Nationale, 13 rue du Four, Paris. (November 1955).

College preparatory mathematics in grades nine through twelve

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A college preparatory course is described by showing how the use of four major mathematical ideas unifies the program of grades nine through twelve.

THERE IS ONE SURE THING about any college preparatory program in mathematics: you are never free of the idiosyncrasies of the learning process. The pupil whom you thought well of will one day ask you what to do next when he meets something like $3=r^3/r^2$; the calculus student is likely to be blocked momentarily when solving something like

$$\frac{3x^2-x}{\sqrt{2x+1}}=0.$$

Such events are so common that we soon learn to accept them, lead the student by the hand over the momentary hurdle, and go on. They are routine. They are, however, about the only things we are completely sure of.

There are many ways to organize a mathematics program, and we are not sure which way—if any way—is the best, but, if one is going to act, one has to reach some conclusions as a basis for action. Two conclusions really form the basis for this college preparatory program.

First, the program should emphasize, as much as the maturity of the students permits, the essential unity of the subject. This means that the vocabulary has to be accurate and modern. There is no course called geometry, or second year algebra, or solid geometry, and every teacher must be thoroughly acquainted with all four years of the subject. It should be noted here that although there is no course called geometry, we do use books called geometry, and books called algebra. We prefer to keep the books relatively conventional and achieve our "integration" by using a syllabus of our own making.

The second conclusion which forms a basis for this program is that a college preparatory program should give the student a mathematical maturity sufficient to cope with the calculus, and, while it is not necessary to give him an introduction to calculus in the secondary school, yet, if he is of the ability usually associated with college preparatory sections—and most college preparatory sections in high schools average around 120 in I. Q.—then we believe it is possible to give him a good introduction to the calculus in grade twelve.

One other point of view that the reader should keep in mind is that the broad outline of a mathematics program that is sound for college preparatory students is also sound for noncollege preparatory groups. The latter may move more slowly, some elaborations in depth of treatment may be omitted, but if the noncollege-preparatory courses are mathematics courses at all, then their general pattern will be the same as the pattern for the college preparatory groups.

It would seem that the best way to make clear the general flavor of the program is by describing the ideas the pupils should be "comfortable with" before they start the calculus. At the almost certain

cost of considerable over-simplification, only four ideas will be traced through the program.

These ideas are: (1) the postulational basis of the subject, (2) the function concept, (3) the one-to-one correspondence between pairs of real numbers and points of the plane, (4) the number system.

Please remember now, that only the progression of these ideas from grade nine up to a calculus course will be described. While our calculus course comes in grade twelve, this is by no means necessary, and the progression of the ideas may well be spread over four years instead of three.

The postulational basis of the subject

The starting point for grade nine is the sentence: The introduction of letters to stand for numbers focuses our attention on the rules for combining numbers. This means that the cummutative, associative, and distributive postulates are explicitly referred to as postulates. It means that equalities are dealt with by using the axioms of addition, subtraction, multiplication, division, and substitution. It means that the student is made aware of the general idea that the solution of an equation consists of using a set of agreements about numbers. Thus, when he gets to the end, the student won't write "$x=2$ *feet*."

The order of operations is presented, not as a necessary fact, but as an agreement that is necessary to avoid ambiguity. The operations with negative numbers are defined. The reason for the definitions is made explicit.

Throughout the ninth grade, the "if-then" pattern of speech is used wherever possible. In some of the better sections it is possible to change the nature of the elements the student is dealing with by introducing some plane geometry. Much of the logical pattern of geometry is already familiar from the words used in the algebra. For example, the notion of converse is explicitly brought out by phrasing the check of the solution of an equation like this: If $2x+1=7$, then $x=3$, and conversely, if $x=3$, then $2x+1=7$.

In grade ten the development of plane geometry proceeds by groups of related theorems with definitions and postulates and undefined terms relevant to the group explicitly set forth. The ultimate logical identity of algebra and Euclidean geometry is mentioned, not pursued. But geometry is not presented as a single, perfect example of a logical development.

The operations with irrational numbers are defined. Exponents and logarithms are defined and developed. The theory of logarithms is touched upon, perhaps at the expense of a few fussy originals in geometry. We define the sine, cosine and tangent of acute angles, and are careful to make plain that these are definitions and that they will eventually be seen as special cases of more general definitions. The role of definitions is emphasized again when the angles between a line and a plane and between two planes are computed. The one-to-one correspondence between pairs of real numbers and points of the plane is postulated and, with the help of this postulate, the course goes on to slope, intercepts, arbitrary choice of axes, locus, and graph.

In grade eleven, geometric and arithmetic language become more and more interchangeable. For example note "the zeros of a function" and "the intercepts on the x-axis," the "value of a function" and "the y-co-ordinate of a point." Euclidean geometry is "completed." The trigonometric functions are redefined in the usual way so that such functions of any angle can be considered. The course does not attempt, at the eleventh grade level, the additional generalization to the trigonometric functions of a number. From the usual definitions there follows the development of some of the elementary identities.

In grade eleven, the theory of logarithms is carried a little farther than it was in grade ten. The division identity is introduced with the $f(x)$ notation and the remainder and factor theorems proved. Imaginary numbers are introduced, the

equality of complex numbers is defined and also the operations with them.

The function concept

In grade nine, the most important single notion associated with the function concept is that of variable. We do not define variable as "a quantified (dummy) symbol in a proposition or designation." We think that about all the student can handle is "a variable is a letter used to stand for any one of a set of numbers."

We do not emphasize the word "function" in grade nine although it gets used some times in class. We talk a great deal about relationships and about dependence. The student makes tables of values and does a little graphing. He learns the distinction between a conditional equation and an identity and gets used to associating sets of numbers which are related by a rule. He becomes thoroughly familiar with the equation as a translation of a verbally stated relationship, with the formula as a convenient dodge for remembering relationships, and with the identity as the end product of the multitude of situations in elementary algebra where the job has been to effect a change in form but not in value.

We are aware of the confusion now existent about the word "function." I know of no book available for secondary schools which defines a function as "a set of ordered pairs, no two of which have the same first element." We were all brought up on Dirichlet's definition: "If y and x are so related that for any value of x in a certain range there corresponds a value of y, then y is said to be a function of x." The books we use certainly use the latter definition and are likely to talk about ax^2+bx+c as the quadratic function instead of the function f: (x, y) given by the rule $y=ax^2+bx+c$. We are at least conscious of the fact that when we write $f(x)=x^2-5x+6$ and say $f(x)$ is the function x^2-5x+6, we are using $f(x)$ in a double sense: it stands for the function and it stands for the value of the function at a particular value x of the independent variable. We don't even know, I suppose because we haven't had the courage to try it out, whether tenth graders can absorb comfortably the phraseology "a set of ordered pairs, no two of which have the same first element."

However this present confusion may come out, here is what we actually do know about the function concept and its related notions in grades ten and eleven.

In grade ten, the notion of domain and range is constantly implicit in our phraseology. For example, in the definition of an identity, the phrase "for which the statement has meaning" receives increased emphasis. The solution of the quadratic equation is dealt with explicitly in the domain of real numbers. I'm afraid we are still guilty of using such terminology as "Express the area of the rectangle as a function of x. For what value of x is the area equal to 96?" The linear function is studied rather thoroughly. The $f(x)$ notation and the $\Delta y/\Delta x$ notation are introduced.

In grade eleven, the polynomial function is studied with major attention to the quadratic and the cubic. Here, of course, range and domain must be explicitly considered, especially when the functions are related to their graphs. The trigonometric functions receive more than a passing nod. Their range and periodicity are looked at, the notion of continuity is intuitively considered for all functions herein mentioned, but especially in connection with $y=1/x$ and $y=\tan\theta$, and in the discussion of the solution of higher degree equations for irrational roots. By the end of the eleventh grade the students are thoroughly familiar with the $f(x)$ notation and with the delta notation.

Analytic geometry

The notion of a one-to-one correspondence between points and real numbers first appears in the development of positive and negative numbers. The extension of this notion to pairs of real numbers and points of the plane begins in grade nine of course,

though not very extensively.

In grade ten, comes the first systematic elaboration of the idea when points, distance between points, lines determined by two points, parallelism, and perpendicularity are analytically considered.

In grade eleven, the conic sections are studied, the parabola in some detail, including extreme values. The other conic sections are developed in less detail. The main object here is the association of the curve with the equation. The curves are defined in terms of distance rather than in terms of the focus and directrix. The translation of axes is touched on slightly, rotation not at all. Careful attention is paid to the range of values for which the functions are defined.

The notion of continuity is associated with the curve. The investigation of asymptotes leads to the language of limits. The general method of closing down on an irrational root by interpolation receives its geometric interpretation. The linear interpolation of a table is associated with the graph of the function which is exhibited by the table.

The number system

When positive and negative numbers are introduced in grade nine, the first questions about a field are introduced: "When we add two numbers, do we always get a number of our number system? When we subtract two numbers, do we always get a member of our number system?" Irrational numbers come in briefly in grade nine.

In grade ten, the irrational numbers are explicitly and consciously added to the number system. Rational approximations to irrational numbers are brought in at this time.

In grade eleven, the systematic investigation of the quadratic equation forces the addition of imaginary numbers to the number system and the introduction of the notion of complex number. Actually this is about all we do with them. We learn how to carry out the operations of addition, subtraction, multiplication, and division with them. We prove that, in a polynomial equation, complex roots occur in conjugate pairs. We define their equality, and point out that the concept of order is irrelevant to them, but we do not introduce Argand's diagram nor their trigonometric representation. We keep our attention focused primarily on real variables, for this is the basic subject matter of elementary calculus.

Let me give a partial summary of the course by the rather negative device of telling briefly what is not done in it. There is no extensive drill on elaborate sets of equations in three unknowns. Something is done with them to be sure, but if they are linear they usually come out of situations where they are best handled by determinants (which do not touch), and if one or more of them is of the second degree, they serve no useful purpose at this stage.

The student is not forced to spend time memorizing proofs of theorems in plane geometry. We develop the theorems in class and expect them to follow the development, but our interest here is twofold. Students should see how a logical sequence develops by taking a definition or two, a postulate or two, some undefined terms, and proving the theorems that follow from these fundamental beginnings. Then we want pupils to be able to use the facts of the theorems and definitions to deal with new situations. But the new situations are relatively simple. Great quantities of fussy originals have been eliminated though the pupil still gets plenty of practice in proving theorems for himself.

Some of the time gained by handling plane geometry this way enables us to include all the three-dimensional work that is needed. Such work occurs in all three grades, nine, ten, and eleven. The so-called logical development of solid geometry has been entirely eliminated. But the pupil must learn how to draw the figures—and draw them well. He learns to "see" the

figures, and how to use the basic numerical relationships.

We do nothing with scales of notation, deMoivre's Theorem, determinants, the focus-directrix definition of the conic sections, and very little with fussy problems involving the relations between the roots and the coefficients of a higher degree polynomial equation. We skip the Law of Tangents, soft-pedal the numerical solution of the triangle, and spend most of our "trig time" on analytic trigonometry. We complete analytic trigonometry in the first six or eight weeks of grade twelve.

I have noted little of an explicit nature about "integrating" the various branches of the subject. Just to remind you of the first "conclusion" mentioned at the start, let me say that what we do could be roughly described as elementary algebra in grade nine. Then, in grades ten and eleven, we have stirred together what is usually known as plane and solid geometry, second year algebra, a little bit of college algebra, trigonometry, and analytic geometry up to recognition of the conic sections.

SUMMARY

The essential unity of the subject should be the major determinant in the selection and organization of the materials for grades nine to twelve. The great flexibility of such an arrangement makes it possible to handle the course like an accordion: it can be stretched out for the slower pupils and compressed for the faster ones without deviating from the main ideas of the subject. Thus, while the program leading up to the calculus as a program for grades nine, ten, and eleven has been discussed, it can be spread over four years without the slightest change except the obvious one of taking more time and of completing the analytic trigonometry. At the end of four years the student is still ready for the calculus.

And, finally, the students thrive on it. The broad sweep of the subject makes sense to them, the gathering up of all the varied ideas of these years into the new framework of the calculus gives these ideas relevance, meaning, and purpose as nothing else has ever done. They are stirred, excited, and pleased. And so are we!

Letters to the editor

The following solution to certain equations proved to be very interesting to my Algebra II class so thought I would pass it on.

If one boy can do a job in A days and another boy can do the job in B days, how long will it take if they work together? The equation for this type of problem is

$$(1) \qquad \frac{1}{x}=\frac{1}{A}+\frac{1}{B}.$$

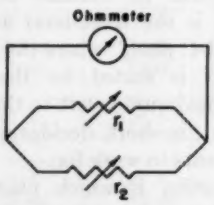
Focal length, image distance, and object distance are related by

$$(2) \qquad \frac{1}{f}=\frac{1}{D_i}+\frac{1}{D_0}.$$

The form of these equations is identical to that for finding the total resistance of resistors in parallel. The total resistance of two resistances in parallel is

$$\frac{1}{R_T}=\frac{1}{r_1}+\frac{1}{r_2}.$$

To solve equations (1) or (2) use resistors whose resistances in ohms are equal to A and B or D_i and D_0. The ohmmeter reading gives the result. (See figure.)



ARTHUR PETERS
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The comparative effectiveness of one-level and three-level assignments in plane geometry

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Many teachers carry out small research projects during the year. This study of the contract plan vs. the one-assignment plan illustrates what can be done by an individual teacher.

The problem

Much has been said and written in favor of differentiated assignments in all high school subjects, but the idea has not had widespread acceptance in plane geometry. Thoughtful teachers recognize that pupils within a classroom differ widely in their rate of learning, but little has been done to meet the problem.

A meaningful course in plane geometry compels the centering of attention upon the learner and his particular difficulties in learning. It compels the teacher to continuously evaluate his materials of instruction—the heart of which is "the assignment."

The traditional assignment that requires all students to learn what is provided at the same rate is still in use. It gives the maximum requirements while the teacher "secretly" decides what is the minimum work accepted. In contrast to this arrangement, is the three-level assignment or "the A-B-C plan," where the minimum requirement is stated by the teacher, while the maximum is left to the student. The student, in short, decides what letter grade he wants to work for.

The question is which plan is more practical and more satisfying to the needs of the majority of the students as determined by their actions and achievements. The following study offers some interesting evidence on this subject.

The experimental populations

Seventy-nine students of plane geometry from Lincoln High School of Wisconsin Rapids, Wisconsin, participated in this study. All were from grade ten. They were divided into three classes, each of which met for fifty-seven minutes, five days a week, and were all taught by the same instructor.

Table 1 gives some of the most important characteristics of the groups. The individuals were grouped upon the basis of similar interests or vocational preferences. Group I was composed of students primarily interested in auto mechanics and nursing, Group II in engineering, and Group III either in commercial or social work.

Algebra grades were the most extensively used source in predicting success in geometry. I.Q. scores obtained on the *Terman-McNemar Test of Mental Ability* were also considered, but were not used as reliable data. No geometry aptitude tests were used. From Table 1 it is evident that these were heterogeneous groups, the range of learning ability being about as great as that found in many such "normal" groups.

TABLE 1

CHARACTERISTICS OF THE GROUPS UNDER STUDY

GROUP	NO. IN THE POPULATION		I.Q. SCORES (TERMAN—FORM C)				GRADE IN FIRST YEAR ALGEBRA			
	Boys	Girls	Low	High	Mean	S.D.	Low	High	Mean	S.D.
I	15	12	84	124	105.8	10.99	79	95	83.6	5.05
II	24	0	89	127	105.9	10.87	75	93	84.7	5.49
III	12	16	90	137	113.9	12.09	74	95	87.1	5.17

THE PROCEDURES

As the school year was divided into quarters, it was convenient to experiment and evaluate certain practices within this block of time. During the first quarter each group used the one-level assignment plan. Although only one assignment was given, not all the students were expected to do all the problems, but each was expected to do as many as he could. Students were graded on the amount of work they had done correctly.

During the second and third quarters, the three-level assignment plan was adopted for all classes. During the last quarter the one-level assignment was reinstated by Group I, while Groups II and III continued with the multiple plan.

Throughout the year all students were given the same weekly and quarterly tests. Projects were also given to students wishing to do extra work. Finally, the *Davis Test of Functional Competence in Mathematics* was administered to all students at the end of the year. It was given expressly to determine individual achievement of the basic mathematics objectives laid down by the Commission on Post-War Plans.

SUMMARY AND RESULTS

To ascertain the relative effectiveness of the two plans of assignments, several specific questions were set up as criteria: (1) Were any significant trends indicated by observing quarterly achievement averages of each group? (2) Were the differences in achievement under the two designs of assignments significant? (3) Were the differences more common to one group than another? (4) Which plan did students seem to prefer?

Table 2 shows the over-all achievement made by each group during the four quarters of the school year as well as other important averages. The most striking thing is the decline in mean averages for the second quarter. While one cannot entirely neglect losses due to adjustment to the "A-B-C" assignment plan,

TABLE 2

MEANS AND STANDARD DEVIATIONS OF ACHIEVEMENT AVERAGES OF THE GROUPS

GROUP		FINAL QUARTER GRADES				FINAL TEST	DAVIS TEST	FINAL GRADE
		1st	2nd	3rd	4th			
I	M	84.1	80.4	79.9	78.2	47.7	119.5	80.4
	σ	5.09	8.15	8.99	10.85	13.51	12.93	7.60
II	M	83.9	83.1	83.8	83.4	54.4	122.8	83.5
	σ	7.53	8.35	5.64	7.16	15.64	12.33	7.01
III	M	86.9	85.7	86.3	85.1	53.5	129.1	85.7
	σ	4.72	5.58	5.89	4.45	13.53	10.59	5.29

TABLE 3

MEANS, STANDARD DEVIATIONS, AND DIFFERENCES IN DAILY PERFORMANCES MADE BY THE "PAIRED" GROUPS

		Daily Av. 1st Quarter	Daily Av. 2nd Quarter	Daily Av. 3rd Quarter	Daily Av. 4th Quarter	Final Test Av.	Final Grade Av.
Group Y	M	87.75	84.05	83.65	83.65	51.15	82.8
	σ	4.15	4.21	4.38	4.88	12.71	4.47
Group X	M	87.55	84.20	83.60	85.05	50.95	82.9
	σ	4.31	5.18	4.60	5.31	13.12	5.25
r		.67	.61	.79	.61	.88	.82
Diff.		.20	−.15	.05	−1.40	.20	−.10
C.R.		.254	−.153	.754	−1.35	.139	−.089

the lower averages seem to indicate that many students were satisfied with doing the minimum requirements of this plan. The group least affected by the change of assignments (as indicated by the standard deviation) was Group II, composed only of boys.

In general, there is evidence that students tend to do more and, perhaps, better daily work, under the one-level assignment plan than under the three-level plan. However, since the former plan stresses the maximum requirements while the latter stresses the minimum requirements needed to get passing grades, better averages under the first plan may well be due to psychological pressure.

Correlations made between quarter achievement scores and the final geometry grades were also statistically significant and revealing. The averages attained during the second and third quarters, the period during which the three-level assignment was used, correlated better with final grades than did averages achieved under the one-level assignment plan by as much as .13. Another way of saying this is that daily averages attained under the multiple assignment plan seem to represent more truly the student's overall achievement as indicated by his final grade than do averages attained under the one-level assignment.

An attempt was made to pair a number of pupils from Group I with an equal number from the combined Groups of II and III. To resort to symbolism, the two new groups are called Group X and Group Y respectively. The pairing was done on the basis of (1) final algebra grades, (2) I.Q. scores, and (3) sex. On this basis it was possible to select twenty reasonably well-matched pairs of pupils. Table 3 shows that comparable results were found in the performances of these groups during the first, second and third quarters of the school year when all students were following similar instructional procedures. A difference is found for the fourth quarter. It was then that pupils of Group X (all members of Group I) used the one-level assignment plan while members of Group Y used the multiple assignment plan. This again indicates that students achieved higher grades under the former plan as was stated earlier. When asked which plan they prefer, students generally had no preference. Some were quick to say the one-level assignment.

It hardly seems necessary to point out that this study was quite inconclusive, yet the evidence presented still seems to indicate that averages obtained under the three-level assignment plan are better predictors of final geometry grades than averages from the one-level plan, although the averages were lower. The results of this study must be used with caution. Nevertheless, it is hoped that mathematics teachers may find these summaries of some use in developing patterns for studying their instructional program.

Progress in mathematics instruction

JOHN R. MAYOR, *Director of AAAS Science Teaching Improvement Program, Washington, D. C.*

Teachers of mathematics and science will be interested in a brief report on the proceedings of the recent Conference on Mathematics Instruction sponsored by the AAAS.

THE NATIONAL PUBLICITY now being given to the importance of the study of mathematics and science in secondary schools must be a source of real satisfaction and inspiration to all teachers of these subjects. That this publicity is so frequently accompanied by statements like that which appeared in a news story by Benjamin Fine in *The New York Times* for November 2, 1956, should not detract greatly from this satisfaction, if this criticism is interpreted properly. This story by Mr. Fine begins "Science and mathematics are the 'weak sisters' in the nation's high school curriculum, educators charged yesterday."

The reported charges of the educators were made at the 21st Annual Educational Conference sponsored by the Educational Records Bureau and the American Council on Education. Actually the "weak sisters" are the victims, not of failure, but of success. We find ourselves in a new age of science and technology, in which secondary school mathematics and science have roles of importance never before conceived even by the most optimistic of us. This new age has been made possible by successes of those who have been in the classrooms of our secondary schools.

That the teaching of secondary school science and mathematics too often does not reflect modern scientific advancements is the basis of a valid criticism. But this criticism cannot appropriately be made a criticism of secondary school teachers, particularly in mathematics. Those teachers even now being graduated from many colleges and universities with a recommendation for a teacher certificate in mathematics have not had courses in mathematics at the undergraduate level which prepare them fully to understand comments on outmoded mathematics, much less to plan their courses to take advantage of constructive criticisms. The mathematics of the undergraduate level in our colleges is probably even more outmoded than that of our secondary schools. It remains to be determined just how "modern mathematics" can most effectively improve our courses at all levels of instruction. Although the criticism cannot be appropriately directed to secondary school teachers, this does not free them from their responsibility to work hard toward the needed corrections. The next few years will provide many opportunities for this.

As an illustration of the genuine interest of scientists in the improvement of instruction in mathematics and science, reference in the remainder of this paper is made to a recent Conference on Mathematics Instruction[1] sponsored by the American Association for the Advancement of Science. The conference provides an illustration of one of the kinds of

[1] A part of this paper is based on a report on the Conference on Mathematics Instruction which appeared in *Science* for December 14, 1956.

assistance that is now, and will be, available to secondary school teachers. The conference also makes clear that a great deal more progress in curriculum is being made or is already under way in mathematics than in the other sciences.

Perhaps the greatest amount of criticism has been in the area of mathematics. Many believe that the lack of interest in the study of science and mathematics in high school may be due, in considerable part, to what they regard to be the low quality of the teaching of arithmetic in the elementary grades. Recognizing the concern about mathematics instruction and the interest of scientists, as well as mathematicians, in the teaching of mathematics, the American Association for the Advancement of Science called the Conference on Mathematics Instruction in Washington early in October 1956. The conference was made possible by a grant from the Carnegie Corporation of New York. The twenty-six participants included representatives of the three professional societies in mathematics, the American Institute of Biological Sciences, the American Institute of Physics, the American Chemical Society, the Office of Education, and leaders in research in applied mathematics, statistics, and psychology. Also invited to participate were teachers of arithmetic and junior and senior high school mathematics.

Reports were presented on activities and plans of the Commission on Mathematics, sponsored by the College Entrance Examination Board; the National Council of Teachers of Mathematics Curriculum Committees at the elementary and secondary school levels; the curriculum activities of the Mathematical Association of America; the University of Illinois experiment in secondary school mathematics; and the Study on Admission to College with Advanced Standing.

In addition to the reports on activities directed toward the improvement of mathematical instruction, there were discussions of a number of related topics, such as what needs to be done at the elementary school level, point of view of scientists on mathematics curriculum, problems in teacher education, and possibilities for and the desirability of better co-ordination of elementary school, secondary school, and college programs. In the course of the two-day discussions considerable attention was given to the need for emergency action as well as long-term study. The need for research in the learning of mathematics, especially in problems of student motivation and questions of readiness at various age levels for the study of science and mathematics, was emphasized.

In recognition of the fact that professional groups in physics, chemistry, and biology, as well as in mathematics, are now engaged in or making plans for curriculum studies, it was suggested that there should be at least one central agency which would be informed of such plans and studies, and be in a position to make reports on progress to other interested persons. Problems related to better co-ordination of instruction in mathematics at the various levels and better co-ordination of professional activities directed toward the improvement of this instruction were considered. The conference pointed out that the National Council of Teachers of Mathematics is the appropriate agency to provide this co-ordination.

The conference asked that support be sought for curricular understanding with secondary schools on minimum mathematics programs. It was concluded that such a program should permit a high school graduate to undertake, as his first course in college, a course in analytic geometry and calculus such as described by the School and College Study on Admission with Advanced Standing. It was then further agreed that the offering of the course in analytic geometry and calculus in high schools is desirable. The need for higher mathematical literacy on the part of all high school students was also recognized.

The work of professional groups presently engaged in the study of mathe-

matics instruction was endorsed by the conference, and these groups were urged to continue their important activities. The conference revealed that there were in progress a substantial number of studies of real promise, that much remains to be done, and that it is important for the recommendations of these groups to be made known widely, and at an early date.

Secondary school teachers of mathematics will be interested in the activities sponsored by professional groups and directed toward the improvement of mathematics instruction at the college level. Reference has already been made to the important work of the Mathematical Association of America. The Social Science Research Council recently published a statement of recommended policies for the mathematical training of social scientists.[2] A report on the teaching of mathematics for engineers was also recently issued by the Joint Committee of the American Society for Engineering Education and the Mathematical Association of America on Engineering Mathematics.[3]

The number of summer institutes for mathematics teachers will be greatly increased during the summer of 1957. Because of the concern of the Congress about secondary school mathematics and science, the National Science Foundation will sponsor a greatly expanded program of institutes. Those institutes sponsored by industry will be continued and in some instances increased in number. One of the best ways for mathematics teachers to meet the challenge of the new mathematics will be through attendance at the institutes.

Teachers of mathematics will also find college teachers of mathematics very ready to assist them in state and local groups. It is not too early for the establishment of state committees on mathematics curriculum. These committees probably can be most effective if they include teachers and administrators, representatives of the state departments of education, mathematicians and scientists in fields which make wide use of mathematics.

[2] *Items*, vol. 9, No. 2, June 1955: "Recommended Policies for the Mathematical Training of Social Scientists."

[3] *Journal of Engineering Education*, vol. 45, No. 8, April 1955.

A DATE TO REMEMBER

Thirty-Fifth Annual Convention

THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

Bellevue-Stratford Hotel
Philadelphia, Pennsylvania
March 27–30, 1957

Edited by Phillip S. Jones, University of Michigan, Ann Arbor, Michigan

Using the history of mathematics in teaching on the secondary school level

by Herta T. Freitag, Hollins College, Hollins College, Virginia, and Arthur H. Freitag, Jefferson Senior High School, Roanoke, Virginia

More than two thousand years ago, Pythagoras lectured on the power of number, meaning only integers and fractions. He reiterated again and again, "Number rules the universe; the essence of all things is number." Then one day he evaluated the diagonal of a unit-sided square. Alas, there was no number as he understood number which gave an answer.

Legend has it that one of his grief-stricken students contrary to the rules of Pythagoras' esoteric school divulged this number fiasco to an outsider. The student was drowned. Students are no longer drowned for breaking the rules of a school. But to know the difference between rational and irrational numbers is as important today as it was then.

Any educator interested in improving the teaching of mathematics must consider a more intelligent use of historical material. In ancient times, man was a local creature. During the Middle Ages he grew in social awareness and became provincial in his outlook. At the onset of modern history, nationality became his important social interest. Today not only do nearly all our problems affect the entire world, but many of them can be understood only in the light of their history. When discussing the uses of historical material in teaching secondary school mathematics, pertinent questions naturally arise. How does the knowledge and proper use of historical material aid both the teacher and learner of secondary school mathematics? Which historical approach will be most beneficial? Should the historical material be presented as part of an integrated course, or should specific units be given separately? Should it be chronological facts, as well as the biographies of the great mathematicians? And above all of these questions, do we have any valid criteria to offer as a basis of selection from these many possible approaches?

The historically-oriented plan

The historical approach to the teaching of mathematics requires that mathematics be presented as an ever-evolving human endeavor. In this "evolutionary approach" plan, an occasional remark about a mathematician, a footnote of an historical nature are in no way adequate. Nor should the history of mathematics be given supplementary status or relegated to the mathematics club. Investigations for determining the value of the integration of this historical material in courses in mathematics should run concomitant with actual teaching under this "evolutionary plan." This latter implies that the complete evolution of an idea may be traced, and

the life of one of its creators should be described.

The human interest factor as well as mathematical content seem the all-important bases upon which the selection of the historical material is to be efficiently integrated. The essence of this suggested method is *flexibility*. The over-all basic principle for the choice of material is a constant concern for the improvement of mathematical learning. That the history of mathematics is inextricably interwoven with the sum total of mathematical knowledge is the basic assumption. Furthermore, there is no one ideal method of teaching. If this approach is to be of educational value, the teacher's judgment must be final.

An analysis of the possible benefits that may be derived from the historical approach

A proper fusion of the history of mathematics with its teaching and learning can only produce desirable results. Some of the possible improvements are in deepening, fostering, and strengthening the outcomes of the traditional approach. Others lie in areas not so easily reached by the usual and ordinary methods. The methods and materials that ought to be used must in the very nature of the problem remain in the form of general suggestions.

The historical method is an asset to the attainment of mathematical objectives of both teaching and learning. It leads to a better understanding of theoretical mathematics and greater technical skill in its application. Mathematics' role in the rise of civilization is appreciatively recognized and this recognition is apt to influence favorably the whole personality of the student. The endeavor to make the history of mathematics an *integral* part of the teacher-learner rapport magnifies the instructor's power of presentation.

Mathematical meanings become more lifelike by portraying the ontogeny of the evolutionary structure of mathematics. One illustration of the ecological basis of this approach is shown in the development of decimal fractions and the invention of an adequate symbolism for its growth. Furthermore, here we see that the historical order is not necessarily the logical one, and the vital part that intuition plays in the realm of human thought. The importance of considering details is recognized through the correlative and cumulative nature of mathematics. It takes its rightful place as a progressively growing phase of man's search for knowledge.

The superiority of the language of mathematics in its terminology and symbolism are better comprehended through this historical approach. This superiority has brought mathematics to its present high order of abstraction. It may be a real challenge to the student to decipher some obsolete symbolism. Mathematical language then becomes understood as a growing part of the evolution of the field, and the interdependence of thought and symbolic communication is shown. The problem of mathematical skills (technical ability) is made up of two components. The first is the proficiency with which factual knowledge is applied. The other is an attitude or desire to perform the required operations. Viewing from the peaks of history the panorama of attempts to record numbers opens the infinite vistas of modern electrical computation.

The ever-changing and fragmentary character of knowledge and understanding is exposed by an historical approach. A continuity in mathematics, a possibility of international collaboration among mathematicians (individuals or groups), and the correlating and spirally-linking tendencies are portrayed.

Trigonometric relationships illustrate the cosmopolitan contributions to mathematics. The Greeks gave us geometric equivalents to $\sin^2\theta+\cos^2\theta=1$, the Law of Cosines, the cosine of the difference between two angles expressed as functions of the separate angles, and the relationship for sine of half an angle. The other Pythagorean relationships were be-

queathed by the Arabs who also supplied the formulas for the sine of twice an angle. The Germans developed the reciprocal relationships between the cosine and the secant, the Law of Tangents, a one-half angle formula, and the area of a triangle equivalent to half the product of any two sides times the sine of the included angle. The country of Lafayette contributed the sine of a treble angle, the tangent and secant of any multiple angle, the nth power of the expression $\cos\theta + i\sin\theta$, and other half-angle formulas. Swiss research produced the formulas for the tangent and cotangent of a double angle. An expansion into an infinite series of the sine and cosine of a multiple angle came from the British.

The invention of the wheel by our remote ancestors illustrates the inception of the spirally-linking trend in mathematics. Much later this led to higher levels of abstraction with theorems about the circle. The Greeks had already analyzed conic sections. Descartes reached new levels of abstraction via the function concept in analytic geometry. Projective geometry continued the crystallization of the family relationships among conic sections.

Historically, pure and applied mathematics developed as mutual correlatives. A list of instances of the two-way influencing interaction between pure and applied mathematics would be impressive in extent and length. Frontier work done out of sheer joy in intellectual curiosity has much later been appropriated for far-reaching practical applications. The non-utilitarian ancient Greeks brought forth the conic sections more than a century before Christ. Almost two millenniums later, Kepler used these curves as the basis of his laws on celestial motion. Ballistics and navigation came next and drew forth from these same wellsprings. And now the orbits of the electrons in the hydrogen atoms are found to be conic section gyrations.

Faith in human dignity is encouraged by acquaintance with some of the creative pioneers in this discipline. Social sensitivity (especially in co-operation and tolerance) and a widened perspective are envisioned with those mathematical ideas underlying the beauteous correlation in nature and art. Realizing the relative insignificance of anyone's accomplishments, however great, humility—the very essence of life—is born.

At this point I can feel the sceptic breathing down my neck. What kind of divine enthusiasm could possibly justify such ideal attainments? But I still believe where there is no vision, a nation perisheth, and so without that divine spark does mathematics.

Some reasons for the possible benefits derived from the evolutionary approach

The contributions of an historical approach are either natural concomitants of the suggested method or of the psychological characteristics of the subject matter itself. Gestalt theory emphasizes the mental-emotional unity of the learner. From a vague integrated configurational response the student should be drawn to differentiating recognitions of particular properties and relationships. Then the final step of the master teacher, a clear and thorough grasp of the whole picture, becomes a work of art!

The suggested plan takes care of the mind as well as the feelings. Field theory considers the individual as part of a still greater whole. Interrogate the past, for the human race is more important than the individual. The warp and woof of mathematics, important as it is, can be observed as a part of the whole realm of human endeavors. Connectionism stresses the need for establishing associational bonds in the learning situation. The historical method associates mathematical ideas with their creators, mathematics with the history of human affairs (be it ever so inhuman). Here is the chance to take drudgery out of drill and review and train that memory.

Let little Gauss provide an illustration of our approach in arithmetic and arithmetical progressions. An annoyed teacher hands out a tough problem to a group of elementary school cherubs. Add up all the numbers from one through a hundred! That will keep them busy for a long time. The little boy, Gauss, beamed and put down his pencil, saying "My answer is 5050." The story does not say how well the teacher reacted to these words of wisdom from the mouth of an infant in arithmetic. But it is fair to surmise that Carl Friedrich Gauss or at least his method will be remembered when there is need for adding a series of numbers or an arithmetic progression sum.

Mathematics by its inherent nature is spiral and cumulative. The circular-ladder type of organizing areas in this field is based on sound psychological considerations. To enrich the review of familiar patterns of thought, the spiral procedure distributes drill, provides that much-needed variety of material, and facilitates the proper spacing of learning. Concept formation is a matter of slow and often arduous growth. The evil of rigid over-compartmentalization is automatically thwarted. The imitative urge of young people to be like their hero is adequately satisfied.

An intelligent use of the past can help us solve our problems more reasonably. Mankind's intellectual heritage unfolds as the common property of all. Scientific and humanistic ideologies are reconciled and unified. The universality of change, the fallibility of the mind, and the present uncertainty of all human knowledge (Gödel's theorem) are glimpsed from this perspective.

A final group of reasons explains why the historical method presents possibilities for setting the stage for new topics. It lends itself effectively to the unit presentation plan.

It now remains to show that this plan increases the efficiency of the teacher. By thoroughly absorbing a background that integrates the history of his field with the teacher-learner conjunction, the real meaning of education pervades this setting. A better and fuller knowledge of the psychological principles dawns upon the instructor. The ontogeny of learning recapitulates its phylogeny. The simple counting of objects to natural numbers, fractions, and then negative numbers is a human trait. Concrete experiences are first encountered before abstractions. Generalization is ordinarily the final step in an investigation. The increase of rigor in mathematics is beaten out from the primal rhythms of the Congo to the Music of the Spheres. As George Sarton remarks, only that individual with his "own inveterate and insatiable curiosity, his constant itching for intellectual adventure" may take this path.

Michael Faraday, an average pupil, was ridiculed by his pedantic and sadistic teacher for his speech defect. Steinmetz, after he had made sacrifices for a new suit, was not allowed to appear on the platform with his graduating class because he was a hunchback. Galileo's gift for independent thought was drowned out by the "magister dixit." Gregor Mendel was reminded by his examiners that he had not sufficiently mastered his subject. Henry Huxley's introduction to formal education left nothing but bitter memories. Madame Curie's life at school was made miserable by a fanatic of nationalism. And what was Henry Poincare's score on the Binet test? The same Poincare who has been called the living brain of rational sciences was rated an imbecile. And yet one more, who should be a favorite with all teachers of mathematics, Evariste Galois who first went to school as if to a prison. Reports on his work were continually mediocre, his conduct, argumentative and dissipated. Twice he failed the entrance examinations to the École Polytechnique. This blindness to recognize native intelligence is inherent in all educational systems. George Sarton puts the whole lesson of this tragic life in, "One can never be too kind to the young;

one can never be too tolerant of their faults, even of their intolerance."

What a challenge and a responsibility for a teacher! He must always be a student intellectually struggling for knowledge and perhaps wisdom. Ingenuity, imagination, and flexibility only can assure his wise timing of historical illuminations. Enthusiasm not to be "just a teacher of mathematics but a master teacher of mathematics" will exert that inspirational influence which may be his immortality. Let us close with Glaisher's conviction that, "I am sure that no study loses more than mathematics by any attempt to dissociate it from its history."

What's new?

BOOKS

College

Descriptive Geometry, Steve M. Slaby. New York, Barnes and Noble, 1956. Paper, xiii+353 pp., $2.25.

Elements of Mathematics (2nd Edition), Helen Murray Roberts and Doris Skillman Stockton. Reading, Massachusetts, Addison-Wesley Publishing Company, 1956. Cloth, x+308 pp., $3.50.

Intermediate Algebra (2nd Edition), Paul K. Rees and Fred W. Sparks. New York, McGraw-Hill Book Company, 1957. Cloth, x+306 pp., $3.90.

Structure of Rings, Nathan Jacobson. Colloquium Publications, XXXVII. Providence, Rhode Island, American Mathematical Society, 1956. Cloth, xii+263 pp., $7.70.

Miscellaneous

Calculus Refresher for Technical Men, A. Albert Klaf. New York, Dover Publications, Copyright 1944, Dover Edition, 1956. Paper, viii+431 pp., $1.95.

Mathematics Magic and Mystery, Martin Gardner. New York, Dover Publications, 1945. Paper, xii+176 pp., $1.00.

Theory of Approximation, N. I. Achieser (translated by Charles J. Hyman). New York, Frederick Ungar Publishing Company, 1956. Cloth, x+307 pp., $8.50.

Trigonometry Refresher for Technical Men, A. Albert Klaf. New York, Dover Publications, Copyright 1946, Dover Edition, 1956. Paper, x+629 pp., $1.95.

BOOKLETS

Money Management, Your Automobile Dollar, Money Management Institute of Household Finance Corporation, Prudential Plaza, Chicago 1, Illinois. 36-page illustrated booklet, 10¢ each.

DEVICES

Direct-O-Percenter, Educational Supply and Specialty Co., 2823 Gage Ave., Huntington Park, California, *or* A. C. Vroman, 367 S. Pasadena Ave., Pasadena, Calif. 11″×14″ cardboard printed with right triangle and scales for teaching percentage problems, 50¢ each, quantity discount.

EQUIPMENT

Math-Master Chalk Board, GAMCO Products, Box 305, Big Spring, Texas. 3½′×4′ framed green boards with either rectangular co-ordinates or polar co-ordinates embossed on surface, $32.50 each, or $29.95 each in lots of two or more.

MONOGRAPH

Providing for Outstanding Science and Mathematics Students, Education Monograph XVI, University of Southern California Press, University Park, Los Angeles 7, California. 111 pp., $4.50.

Edited by Lucien B. Kinney, Stanford University, and Dan T. Dawson, Stanford University, Stanford, California

Plato shows in the eighth at Guilford

by Joseph P. Sabo, Guilford Public Schools, Guilford, Connecticu

Plato actually did more than show at Guilford. With some of the students he was a winner and with others he found his place. This all happened in one eighth grade mathematics class and grew out of a study unit of geometry.

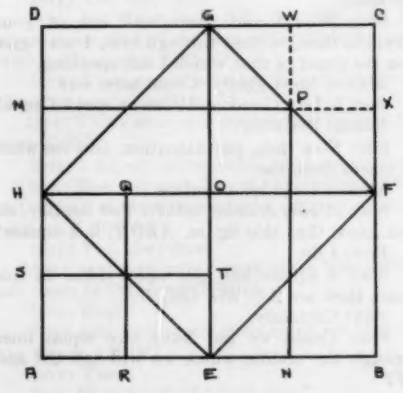
This top section of a group of four classes had already let people in the school know they were inquisitive. They proved their point again during a class discussion of mathematics in ancient Greece. Several students became involved in a supplemental study of mathematics during this time.

It was soon brought to the attention of the class that mathematics was mentioned in the literature of that time by a philosopher named Plato. Very shortly, Plato "showed" in the classroom. A copy of the *Meno* had been obtained, and the section involving geometric terms was read and discussed. A group of students decided to have the entire section mimeographed so that each student in the class could have a copy. Unfortunately, it was also decided, this fellow Plato had confused things a bit in his writing. His Socrates had not only confused Meno and his boy, he had also confused most of the class. Two girls, Faith Kaltenbach and Marshall Pope, decided to rewrite the dialogue so that it could be more easily understood.

The dialogue was then mimeographed and discussed in class. No drawing came with the translation, and so three boys, Robert Hiller, Chester Guiles, and Paul Chello, went to work to create one that Socrates apparently had used. (See Figure 1.)

Special meetings of this group were held for suggestions and corrections of the project. More work followed, and reports were given. Dates were checked regarding the time of Plato's writing and what was known about geometry at that time. Certain mathematical terms were traced to their origin to determine the validity of present translation. A decision was made to use present-day language for a modernized version. A new beginning and ending were worked out. The final drawing used was devised independently of those occasionally found with some translations of

Figure 1



the *Meno*. The figure was labelled and correlated to the dialogue.

Gradually, as the work progressed towards the finish and the many details were ironed out, a sense of pride in the forthcoming result became evident. Everyone was anxious to read the finished product. Finally the text, as given below, was concluded. We here at Guilford are pleased that Plato decided to "show" when he did. He caused many questions to be raised, was responsible for many hours of independent work, and stimulated many group activities. He was welcomed, and we hope he returns.

MENO: Greetings, Socrates. I am most happy that you were able to visit with me today.

Soc: Greetings, Meno. What topic shall we discuss today?

Meno: It matters not. Have you been considering any special ideas of late?

Soc: Yes, I believe that a person in his subconscious mind knows things of which he is not aware. I mean that a person might know the answers to questions, say pertaining to mathematics, and really not realize that he does. Do you agree with me?

MENO: I cannot say that I do, Socrates.

Soc: Suppose I were to question someone on the subject of geometry. I would ask a few fundamental questions concerning basic geometric forms and perhaps lead up to a conclusion of which many students of geometry are aware. Now, perhaps the person questioned is not. Do you think this person might really know the answers to such questions?

MENO: No, I do not!

Soc: Seeing that you do not believe me, I shall attempt to prove it to you by using one of your servants as an example.

MENO: Please do. This should be very interesting.

Soc: Would you please call one of your servants then, so that through him, I may give you the proof of that which I am speaking.

MENO: Most gladly. Come here, boy.

Soc: Is he a Greek and does he speak Greek?

MENO: Perfectly.

Soc: Now then, pay attention, and see what he learns from me.

MENO: I will be attentive.

Soc: (*Using drawing tablet.*) Tell me boy, do you know that this figure, $AEOH$, is a square?

BOY: I do.

Soc: A square has four equal sides. In this figure they are AE, EO, OH, HA.

BOY: Certainly.

Soc: Could we not have two equal lines through the middle which we will call QR and ST?

BOY: Yes, you're correct.

Soc: (*Not pointing to any specific figure.*) Could there be other squares like this, either larger or smaller than this one?

BOY: Certainly there could.

Soc: Now if this side, AE, were two feet long and this side, EO, two feet long, how many square feet would there be in the figure $AEOH$? Look at it this way. Suppose this figure had side AE two feet in length, and side EO only one foot in length, would the area of $AEOH$ be other than two times one or two square feet?

BOY: It would not.

Soc: But since EO is two feet and not one foot, is square $AEOH$ any different from twice two square feet?

BOY: No.

Soc: Does this square then have an area of twice two square feet?

BOY: Yes, it does.

Soc: How many square feet is it?

BOY: Four square feet, Socrates.

Soc: Can there be another square twice as large as this one with all of its sides equal?

BOY: Yes, I am sure there could.

Soc: How many square feet will there be in this other square?

BOY: Eight square feet.

Soc: Try to tell me how long each side of this square $ABCD$ will be. Now in this area, $AEOH$, the side AE is two feet long. What is the length of side AB which is AE extended to point B?

BOY: It is plain, Socrates, that it will be twice the length of AE which is a two-foot side on the first square.

Soc: You see, Meno, that I teach this boy nothing, but only question him. The boy thinks he knows what the length of a side is from which an area of eight square feet is produced. Do you think he knows what he's talking about?

MENO: I do.

Soc: But does he really understand?

MENO: Certainly not.

Soc: Yet, he thinks he does from the idea of a double quantity.

MENO: Yes, that is right.

Soc: Now, listen closely as we proceed with the problem. (*Socrates turns to the boy.*) Now, boy, speak to me. You say that when a line doubles in length, there is produced an area doubled in size. I'm referring to a square having an area twice the size of four square feet. Do you still think this will be from the double line, AB?

BOY: I do.

Soc: Does this line, BC, become double of BF if we extend it from F to Point C?

BOY: Yes, surely.

Soc: Now from this line AB you say there will be an area of eight square feet, if there are four such sides to this square figure.

BOY: I do.

Soc: Now let us draw from it the other three equal sides. Would the area of figure $ABCD$ be other than eight square feet?

BOY: Not at all.

Soc: Are there four small squares within this large square *ABCD*, each of which is equal to four square feet?

Boy: Yes.

Soc: How large is the whole area, *ABCD*? Is it four times as large as one of the small squares?

Boy: Certainly.

Soc: Is that not twice the area *AEOH* which was taken four times?

Boy: No.

Soc: How many times then?

Boy: Four times the area of *AEOH*.

Soc: Then from two equal lines which are double in length, an area four times as large is produced.

Boy: That is true.

Soc: Four times four is sixteen, isn't it?

Boy: Yes.

Soc: Then from what line is to be drawn a square with an area of eight square feet? Isn't it from this line, *AB*?

Boy: Yes, I think so.

Soc: And from half of this line *AB*, which is *AE*, there is produced an area of four square feet.

Boy: There is.

Soc: Now, isn't the figure *ADBE* eight square feet since it is twice as large as this figure, *AEOH* and half as large as *ABCD*?

Boy: Certainly.

Soc: Will the square then not be from a line longer than *AE* but shorter than *AB*?

Boy: That is the way it looks to me.

Soc: What you say is correct, boy. And is not *AE* two feet long and *AB* four feet long?

Boy: Yes, that is right.

Soc: The line then from which an area of eight square feet is produced must be greater than two feet and less than four feet.

Boy: It must be.

Soc: Tell me then how long you think it is.

Boy: Three feet long.

Soc: If it is three feet long, we will add one half of *EB* to *AE* and this will be a line *AN* which is three feet long. For *AE* is two feet and the extra part, *EN*, is one foot long. And likewise side *AH* is two feet and by adding half of *HD* to *AH* we have a side *AM* of three feet.

Boy: It is so.

Soc: So if this line *AN* is three feet long and *AM* is three feet long, the square *ANPM* has an area which is three times three square feet. (*Socrates draws line MX and NW forming AMPN.*)

Boy: Certainly.

Soc: How many square feet are three times three square feet?

Boy: Nine square feet.

Soc: But how many square feet should the original doubled area be?

Boy: Eight square feet.

Soc: Then from two three-foot sides you cannot form an area of eight square feet?

Boy: You cannot.

Soc: But how long must the line be? Try to tell me exactly. If you cannot give me the length in numbers, at least point out from what line it may be drawn.

Boy: But Socrates, I really don't know.

Soc: Do you see, Meno, how this boy is proceeding with the problem? At first he didn't know what the line was which produced an area of eight square feet; in fact, he doesn't know yet; but he thought he knew and he answered boldly, as any person who really knew would. He didn't think he would give the wrong answer, but now he is completely confused and doesn't even think he knows.

Meno: Everything you say is true, Socrates.

Soc: Watch this! Even with his difficulty he will begin to learn. Listen closely as I ask him further questions and the boy presents his answers. (*Socrates turns to the boy.*) Now boy, do you understand that this area *AEOH* is four square feet?

Boy: I do.

Soc: Now we will apply to it this other area *EBFO* which is equal to it.

Boy: All right, proceed.

Soc: And we will also apply to it this area *FOGC* which is the same as each of the first two areas mentioned.

Boy: Very well.

Soc: Now we add this last area *GOHD*, likewise equal to the others, which will complete the large square *ABCD*.

Boy: Very well.

Soc: Will these areas be anything else but four equal areas?

Boy: No.

Soc: Well, then, how much larger is the area *ABCD* than any one square, for example *AEOH*?

Boy: Four times as large.

Soc: But we only wanted one that was twice as large, or have you forgotten?

Boy: No, I remember that very well.

Soc: Follow this closely. (*Socrates draws lines from H to E, from E to F, from F to G, and from G to H.*) Will not these lines cut each of the small squares such as *AEOH* in half?

Boy: Yes, they will.

Soc: Are these lines *HE*, *EF*, *GH*, and *FG* equal in length and do they not include the area within the square *EFGH*?

Boy: They are equal and they do make up the area inside that square.

Soc: Think now, and tell me how large is the area *EFGH*.

Boy: I do not understand what you mean.

Soc: Has not each line of the square *EFGH* cut off half of these four smaller areas or squares, such as *AEOH*?

Boy: Yes, they have.

Soc: And how many such areas like *AEOH* are there in this area *EFGH*?

Boy: Four.

Soc: And how many such areas are there in the figure *AEOH*?

Boy: Two.

Soc: How much of two is four?

Boy: Twice as much.

Soc: How many square feet then does the area *EFGH* have?

Boy: Eight square feet.

Soc: Drawn from what line?

Boy: From this line *HE*.

Soc: You mean from the line starting at *H* and ending at *E* cutting diagonally across the area *AEOH* of four square feet?

Boy: That's right, that line *HE*.

Soc: Now the Sophists call such a line a diagonal, so that from this diagonal there can be produced an area twice as large as ours.

Boy: That is correct.

Soc: Do you remember, boy, that *AE*, a side of two feet, produced an area of four square feet; and *AN*, a side of three feet, produced an area of nine square feet; and *AB*, a side of four feet, produced an area of sixteen square feet?

Boy: Yes, I remember those things.

Soc: Then can we assume correctly that the length of the diagonal line *HE* is a number, which when multiplied by itself, gives an answer of eight? This then is equal to the area *EFGH* of eight square feet.

Boy: Yes, but I do not know what the number is, Socrates.

Soc: The number is 2.828, boy, and therefore *HE* is 2.828 feet long. So you see, Meno, what I said was correct. The boy, in answering my questions, has led up to a conclusion of which students of geometry are aware.

Meno: I do not know what you are talking about, Socrates. What is this conclusion at which he has arrived?

Soc: He has arrived at the conclusion, that the square on the diagonal *HE* is equal to the sum of the squares of the other two sides of figure *AEH*. We have really brought our attention to this right triangle *AEH* with hypotenuse *HE*. This conclusion, as you no doubt realize, applies to all right triangles.

Meno: I must admit that you have proved to me all that you started to prove and more. Now that I think about it, I do remember the relationship between the sides of a right triangle. You're clever, Socrates. I never realized what you were leading up to.

Soc: And so you see, Meno, that in the subconscious mind a person knows and can recall things that he doesn't know he knows. This boy who in the end has told us this thing, had no idea that he could do it.

Meno: I have learned much today. Thank you, Socrates. Goodbye and visit me again.

Soc: Goodbye to you, Meno.

Letters to the editor

In my trigonometry class we have been studying the polar co-ordinate system with a great amount of interest. In fact, one of the better students is co-ordinating his mathematical knowledge with poetry writing in one of his English classes. Below is his poem concerning the same. We thought that it might be used in some issue of The Mathematics Teacher.

The Beauty of the Sine Curve in the
Polar Co-ordinate System
or
Graphic Problems

The sine curve unending, a wonderful line,
Goes on for a reason, my thoughts can't divine.
It loops and crosses without rhyme or reason
To points quite far from the one I envision.

Two-leaf, four-leaf, hundred-leaf roses,
Are drawn by my pen from the scheme it composes,
But no matter the function, any curve whatsoever,
The grade on my graph always ruins the endeavor.

—Verne Griffith, Senior
Okmulgee High School
Okmulgee, Oklahoma

Very truly yours,
John W. Towry
Chairman, Mathematics Department
Okmulgee High School
Okmulgee, Oklahoma

P.S.: I have thirteen boys who are studying one night a week. We intend to work through Herberg's *Elementary Mathematics Analysis*. This is voluntary and no credit will be given, but it is one way we let the better students work further, under guidance. They have named themselves "The Sons of Napier."

• MEMORABILIA MATHEMATICA

Edited by William L. Schaaf, Brooklyn College, Brooklyn, New York

Our favorite quotations . . .

About the beginning of World War I there appeared a serious volume by Robert Moritz called *Memorabilia Mathematica, or the Philomath's Quotation Book*. It was this unique collection of nearly 2200 quotations about mathematics—classified under some twenty-odd headings—which in some small measure at least suggested the idea and the title of this department. A worthy successor to this celebrated book may one day appear; indeed, we sincerely hope so. Meanwhile, we humbly submit a few of our own favorite quotations about mathematics, culled from here and there as the years have gently slipped by. Some of them may have appeared in these pages before—we hope not too many. Like all excerpts, they are apt to lose something when rudely detached from their original context. Yet we feel that each of them has a message to convey. And along with this message there goes the hope that an all too brief vignette may on occasion send the reader scurrying to the original source for further inspiration or enlightenment.

On the prestige of mathematics

Archimedes will be remembered when Aeschylus is forgotten, because languages die and mathematical ideas do not. "Immortality" may be a silly word, but probably a mathematician has the best chance of whatever it may mean.—G. H. HARDY, *A Mathematician's Apology* (1940), p. 21.

. . . in pure mathematics we have a great structure of logically perfect deductions which constitutes an integral part of that great and enduring human heritage which is and should be largely independent of the perhaps temporary existence of any particular geographical location at any particular time. . . . The enduring value of mathematics, like that of the other sciences and arts, far transcends the daily flux of a changing world. In fact, the apparent stability of mathematics may well be one of the reasons for its attractiveness and for the respect accorded it in a world wherein security is so elusive.—M. RICHARDSON, *Fundamentals of Mathematics* (1941), p. 463.

Man has never been a particularly modest or self-deprecatory animal, and physical theory bears witness to this no less than many other important activities. The idea that thought is the measure of all things, that there is such a thing as utter logical rigor, that conclusions can be drawn endowed with an inescapable necessity, that mathematics has an absolute validity and controls experience—these are not the ideas of a modest animal. Not only do our theories betray these somewhat bumptious traits of self-appreciation, but especially obvious through them all is the thread of incorrigible optimism so characteristic of human beings.—P. W. BRIDGMAN, *The Nature of Physical Theory* (1936), pp. 135–136.

As an Art, Mathematics has its own standard of beauty and elegance which can vie with the more decorative arts. In this it is diametrically opposed to a Baroque art which relies on a wealth of ornamental additions. Bereft of superfluous addenda, Mathematics may appear, on first acquaintance, austere and severe. But longer contemplation reveals the classic attributes of simplicity relative to its significance and depth of meaning.—D. E. LITTLEWOOD, *The Skeleton Key of Mathematics* (1949), p. 12.

A mathematician, like a painter or a poet, is a maker of patterns. If his patterns are more permanent than theirs, it is because they are made with *ideas*. A painter makes patterns with shapes and colours, a poet with words. A painting may embody an "idea," but the idea is usually commonplace and unimportant. In poetry, ideas count for a good deal more; but, as Housman insisted, the importance of ideas in poetry is habitually exaggerated. . . . The poverty of ideas seems hardly to affect the beauty of the verbal pattern. A mathematician, on the other hand, has no material to work with but ideas, and so his patterns are likely to last longer, since ideas wear less with time than words.—G. H. HARDY, *A Mathematician's Apology* (1940), pp. 24–25.

On the nature and scope of mathematics

Very few people realize the enormous bulk of contemporary mathematics. Probably it would be easier to learn all the languages of the world than to master all mathematics at present known. The languages could, I imagine, be learnt in a lifetime; mathematics certainly could not. Nor is the subject static. . . . To keep pace with the growth of mathematics, one would have to read about fifteen papers a day, most of them packed with technical details and of considerable length. No one dreams of attempting this task.

The new discoveries that mathematicians are making are very varied in type, so varied indeed that it has been proposed (in despair) to define mathematics as "what mathematicians do." Only such a broad definition, it was felt, would cover all the things that might become embodied in mathematics; for mathematicians today attack many problems not regarded as mathematics in the past, and what they will do in the future there is no saying.—W. W. SAWYER, *Prelude to Mathematics* (1955), p. 11.

We may summarize . . . the fundamental characteristics and limitations of mathematics as follows: mathematics is ultimately an experimental science, for freedom from contradiction cannot be proved, but only postulated and checked by observation, and similarly existence can only be postulated and checked by observation. Furthermore, mathematics requires the fundamental device of all thought, of analyzing experience into static bits with static meanings. In this respect mathematics fails to reproduce with complete fidelity the obvious fact that experience is not composed of static bits, but is a string of activity, or the fact that the use of language is an activity, and the total meanings of terms are determined by the matrix in which they are embedded.—P. W. BRIDGMAN, *The Nature of Physical Theory* (1936), p. 58.

We are concerned to understand the motivation for the development of pure mathematics, and it will not do simply to point to aesthetic qualities in the subject and leave it at that. It must be remembered that there is far more excitement to be had from creating something than from appreciating it after it has been created. Let there be no mistake about it, the fact that the mathematician is bound down by the rules of logic can no more prevent him from being creative than the properties of paint can prevent the artist. . . . We must remember that the mathematician not only finds the solutions to his problems, he creates the problems themselves.—A. H. HEAD, *A Signpost to Mathematics* (1951), p. 19.

On the source of mathematical ideas

The main sources of mathematical invention seem to be within man rather than outside of him: his own inveterate and insatiable curiosity, his constant itching for intellectual adventure; and likewise the main obstacles to mathematical progress seem to be also within himself; his scandalous inertia and laziness, his fear of adventure, his need of conformity to old standards, and his obsession by mathematical ghosts.—GEORGE SARTON, *The Study of the History of Mathematics* (1936), p. 16.

For, in mathematics or symbolic logic, reason can crank out the answer from the symboled equations—even a calculating machine can often do so—but it cannot alone set up the equations. Imagination resides in the words which define and connect the symbols—subtract them from the most aridly rigorous mathematical treatise and all meaning vanishes. Was it Eddington who said that we once thought if we understood 1 we understood 2, for 1 and 1 are 2, but we have since found we must learn a good deal more about "and"?—R. W. GERARD, "The Biological Basis of Imagination," *Scientific Monthly*, June 1946, p. 479.

Men of science belong to two different types —the logical and the intuitive. Science owes its progress to both forms of minds. Mathematics, although a purely logical structure, nevertheless makes use of intuition. Among the mathematicians there are intuitives and logicians, analysts and geometricians. Hermite and Weierstrass were intuitives. Riemann and Bertrand, logicians. The discoveries of intuition have always to be developed by logic."—ALEXIS CARREL, *Man the Unknown* (1935).

. . . we love to discover in the cosmos the geometrical forms that exist in the depths of our consciousness. The exactitude of the proportions of our monuments and the precision of our machines express a fundamental character of our mind. Geometry does not exist in the earthly world. It has originated in ourselves. The methods of nature are never so precise as those of man. We do not find in the universe the clearness and accuracy of our thought. We attempt, therefore, to abstract from the complexity of phenomena some simple systems whose components bear to one another certain relations susceptible of being described mathematically. —ALEXIS CARREL, *Man the Unknown* (1935), p. 8.

On the learning and teaching of mathematics

Only the mathematically minded can really teach mathematics; and it takes a great deal of

mathematics to teach any mathematics well.—C. E. Van Horn, *A Preface to Mathematics* (1938), p. 75.

Our school curricula, by stripping mathematics of its cultural content and leaving a bare skeleton of technicalities, have repelled many a fine mind.—Tobias Dantzig, *Number, the Language of Science* (1930), p. vii.

Then I have more than an impression—it amounts to a certainty—that algebra is made repellent by the unwillingness or inability of teachers to explain why we suddenly start using *a* and *b*, what exponents mean apart from their handling, and how the paradoxical behavior of + and − came into being. There is no sense of history behind the teaching, so the feeling is given that the whole system dropped down ready-made from the skies, to be used only by born jugglers. This is what paralyzes—with few exceptions—the infant, the adolescent, or the adult who is not a juggler himself.—J. Barzun, *Teacher in America* (1945), p. 82.

Why should such fear of mathematics be felt? Does it lie in the nature of the subject itself? Are great mathematicians essentially different from other people? Or does the fault lie mainly in the methods by which it is taught?

Quite certainly the cause does *not* lie in the nature of the subject itself. The most convincing proof of this is the fact that people in their everyday occupations—when they are making something—do, as a matter of fact, reason along lines *which are essentially the same as those used in mathematics:* but they are unconscious of this fact, and would be appalled if anyone suggested that they should take a course in mathematics.

. . .

The fear of mathematics is a tradition handed down from days when the majority of teachers knew little about human nature and nothing at all about the nature of mathematics itself. What they did teach was an imitation.—W. W. Sawyer, *Mathematician's Delight* (1946), pp. 1–2.

In conclusion, we cannot resist adding a few random quotations—perverse if you will, or provocative—but surely not uninteresting.

To ask whether a child has an aptitude for mathematics is equivalent to asking whether he has an aptitude for reading and writing.—C. A. Laisant

Nothing is less applicable to life than mathematical reasoning. A proposition in mathematics is decidedly false or true. Everywhere else the true is mingled with the false.—Mme. de Stael

There are four subjects which must be taught: reading, writing and arithmetic, and the fear of God. The most difficult of these is arithmetic.—Edward Shanks

The nearer man approaches mathematics the farther away he moves from the animals.—Stanley Casson, *Progress and Catastrophe*

The science of calculation also is indispensible as far as the extraction of the square and cube roots; Algebra as far as the quadratic equation and the use of logarithms are often of value in extraordinary cases: but all beyond these is but a luxury; a delicious luxury indeed; but not to be indulged in by one who is to have a profession to follow for his subsistence.—Thomas Jefferson, from an unpublished letter (1799).[1]

[1] See Robert Oppenheimer, *Science*, III (April 14, 1950), 374.

Science and mathematics

Let me tell you about some of these other models that computers are using every day. In my part of the General Electric Company, for instance, we make jet engines. To actually carry a newly conceived jet engine only as far as the flyable prototype—that is still miles ahead of a mass production model—may cost 10 or 20 million dollars. If we build the wrong one, our competitor takes over the business. If we and our competitor both build wrong ones maybe a bigger competitor in Europe will take over everything!

Not only do we have computers making such models on paper, but we have computers telling each other how to do the job. We in Ohio can send a message—in the number language computers understand—to airplane manufacturers on the Pacific Coast who happen to have identical computers. The computers out there can put our mathematical model for an engine (a theoretical, unbuilt, untested jet engine) into their mathematical model of an airplane (a theoretical, unbuilt, untested airplane) and fly that airplane with its engine until the numbers tell us we have the best design. This is not a dream for the future. We've done it. It's a working fact.—*Taken from "The Language Engineer" by H. R. J. Grosch*, The Saturday Review, *December* 1, 1956.

Reviews and evaluations

Edited by Richard D. Crumley, Iowa State Teachers College, Cedar Falls, Iowa

BOOKS

Analytic Geometry, H. Glenn Ayre and Rothwell Stephens. Princeton, New Jersey, D. Van Nostrand Company, Inc., 1956. Cloth, viii +224 pages, $3.85.

The outstanding feature here is the simultaneous treatment of the co-ordinate geometry of two and three dimensions throughout much of the book. The authors claim that this "will greatly enrich the student's understanding of the analytic method in geometry, give him a clearer insight into the nature of a co-ordinate system, and show him the role that analogy and generalization play in the discovery and development of mathematics."

The idea of the simultaneous treatment of the geometry of two and three dimensions appeals to the reviewer particularly in the discussion of direction and of the equations of a line. As the authors state, an added advantage of this type of presentation is that it avoids placing the material on three dimensions late in the text where it may not be reached for lack of time. On the whole, the authors handle this simultaneous treatment well. There are, however, a few places where material about both two and three dimensions is mixed together in the same paragraph, and these places may be confusing to the student. Along the same line, the reviewer is of the opinion that a set of exercises should not contain problems about both two and three dimensions unless it is a review at the end of a chapter.

The authors give only a brief treatment of the general equation of the second degree in both two and three dimensions to save space for an introduction to the derivative, a chapter on empirical equations, and a brief introduction to vectors. The reviewer agrees with this selection of topics. The introduction to vectors is too brief, however, and more applications of vectors should be presented.

There are some places in the text where terms are used carelessly: in several places, an equation is labeled with the name of the corresponding curve instead of being called the equation of the curve; on page 70, the word "system" is used when "members of the system" is meant; on page 171, the authors discuss "the parametric equations of a bullet fired from a gun." There are other weak spots in the text. On page 45, to derive the equations of a line which has one or more of its direction numbers equal to zero, the authors start out with the symmetric form of the equations of the line. This form simply does not apply in such cases, and one must derive the special form from the parametric form. On page 95, the equation of the radical axis of two tangent circles and of two nonintersecting circles is derived from an equation based upon two circles which intersect in two points. On page 108, it is stated that a certain general equation can always be changed to one or the other of two type forms; the very next sentence describes the cases in which this cannot be done. On page 128, the treatment of the rectangular hyperbola with the co-ordinate axes as asymptotes is inadequate. On page 178, the student is asked to obtain the equations of the epicycloid from those of the hypocycloid by an unconvincing mechanical substitution; he would gain more by a complete derivation. (Moreover, there are two unfortunate typographical errors in the problem.)

The typography and format are for the most part good. It would help the reader to have the examples set off in some manner, say by type of a different size. In some places, particularly §12-5, the titles of subparagraphs fail to stand out. The figures are all clear except Figures 7-5 where a few projecting lines would be of great help.

The sets of exercises are in general adequate. The authors have included a number of applied problems. One particularly good feature of the exercises is the inclusion of a review set at the end of each of the first eight chapters.—*Franklin C. Smith, College of St. Thomas, St Paul, Minnesota.*

Analytic Geometry and Calculus, Thurman S. Peterson. New York, Harper and Brothers, 1955. Cloth, ix+456 pp., $5.50.

The analytic geometry portion of this book is rather abbreviated; for example, axes are neither rotated nor translated. The preface states that the student who omits plane analytics and begins with Chapter 3 on limits is not "handicapped as far as the continuity of the discussion is concerned" (although the added experience in working with functions would, of course, be desirable).

There are several welcome features of the book: a chapter on curve-tracing; careful treatment of certain integrals which lead to logarithms; a clear statement of one form of the Fundamental Theorem of the Integral Calculus (based, however, on the definition of the integral as an antiderivative). By and large, this book is a fairly typical representative of the "cookbook" style of text which has long been in common use; it slights introductory explana-

tions and heuristic motivations, but stresses "recipes" for working standardized lists of problems.

This reviewer believes that in an elementary text, rigorous proofs should take second place to sweep and vision, but there seems to be no such compensating gain for the sacrifice of rigor throughout texts of this type. See, as typical examples:

(a) The definition of "critical value" on page 89, which is contradicted by the "Note" on the same page.

(b) Misleading statements about the scope of Newton's method; for example, the implication (on page 182) that it gives only irrational roots.

(c) Theorem 1, on page 312. "A power series may be differentiated term by term for all values of the variable within its interval of convergence." (But whether the result converges, and if so, to the desired function, is not stated.)

(d) Hints that formal methods frequently fail are relatively few. For example, it is stated on page 376 that certain partial derivatives must be zero at a minimum or maximum point, but every conical roof suggests an exception.

Despite these criticisms, it seems fair to say that this book is a worthy member of the "manipulative" school of mathematics texts, and is far more readable and complete than many of its fellows.—*Carl H. Denbow, Ohio University, Athens, Ohio.*

Higher Arithmetic, Virgil S. Mallory and Kenneth C. Skeen. Chicago, Benjamin H. Sanborn and Company, 1955. Cloth, v+418 pp., $3.20.

The text does an excellent job in covering consumer mathematics topics such as insurance, social security, small loans and installment buying, and domestic budgets. The aim of the book is to satisfy a need for a course in socially useful arithmetic. In this respect the authors have achieved their goal. This text could be used profitably for a course in consumer mathematics at any level in the junior or senior high school. It would be well received in many areas by classes in adult education.

Physically the book is attractive, well bound, and small. Its small size is refreshing when compared to many of the oversized texts which have been coming out in recent years; yet, the print is large and readable, the explanations complete, and there are plenty of exercises.

A few minor criticisms can be mentioned. In the chapter on small loans and short-term loans, the amount borrowed is called the principal, and the amount of a loan is defined as the sum of the principal and the interest. This terminology might cause some confusion. Later in this chapter there are some very realistic exercises for determining the rate of interest and the amount of interest in installment buying examples. However, there are a few exercises similar to the following:

A furniture store advertised a table for $16.50 cash, or for $18.50 on the easy payment plan. On this plan the purchaser paid $2.50 down and $4 a month until the price of the table was paid. Find the rate of interest charged.

The reviewer has never seen an ad in which more than one price was listed on an item of merchandise.

Under the section on Bank Discounts there is a statement that interest on short-term bank loans is usually paid in advance. Recent experience indicates that when a person goes to a bank to borrow, say, $200, he walks out with $200. This is not paying interest in advance.

A well taught course using this book as a text should quiet many of the critics of mathematics teaching who claim that the mathematics taught does not meet the needs of a great many of the pupils. If the text is used as a terminal mathematics course it should leave pupils with a good attitude toward mathematics.—*Warren J. Thomsen, Western Illinois State College, Macomb, Illinois.*

INSTRUCTIONAL MATERIALS

Logarithm and Trigonometric Function Tables, On Card (No. 592), W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago 10, Illinois. $8\frac{1}{2}'' \times 11''$ laminated card; sold only in packages of 25, $3.50 per package.

A four-place table of the trigonometric functions appears on one side of the card and a four-place table of logarithms appears on the opposite side. The card is punched with three holes for a standard three-ring notebook binder. Rows of figures in each table are printed alternately in black and red.

The card, laminated with clear plastic, is a desirable student aid. The printing is clear and large enough for easy reading.—*Richard D. Crumley, Iowa State Teachers College, Cedar Falls, Iowa.*

Models for Teaching, The Visual Instruction Bureau, Division of Extension, University of Texas, Austin 12, Texas. 40-page illustrated booklet prepared by Martha F. Meeks, $1.00 postpaid.

This booklet contains descriptions of various types of models—exact models, reduced models, cut-away models, and mock-ups—along with general suggestions for their use. Many special techniques are illustrated for the construction of models, and sources of commercial models are listed.

The booklet is well written and illustrated. Information of this kind has been very scattered, and this publication meets a real need of teachers.—*Richard D. Crumley, Iowa State Teachers College, Cedar Falls, Iowa.*

A column of unofficial comment

The untapped reservoir of potential scientific manpower

by Kenneth E. Brown, U.S. Office of Education, Washington, D.C.

"It is evident that the United States is wasting its intellectual resources at the rate of approximately 200,000 18-year-olds a year. These are the young people with college ability who terminate their education upon graduation from high school." This is the conclusion of Charles C. Cole after a study of the manpower situation.[1] The study, which received financial support from the National Science Foundation, was done under contract with the College Entrance Examination Board.

In *Encouraging Scientific Talent*, Mr. Cole reviews many of the studies that have been made on the number of capable high school graduates who fail to go to college. Although there are differences as to numbers, all studies indicate that there is a large pool of undeveloped scientific talent in those high school graduates who fail to continue their formal academic training.

How can we tap this pool? Dr. Cole states "there is no single dramatic solution to the problem of how to stop wasting intellectual resources." Many persons have suggested more financial aid to college students. In fact, one of the reasons for Dr. Cole's study was to secure data on the desirability of more college scholarships. He concludes that we should have "at least 100,000 scholarships immediately." Even though evidence seems to indicate that perhaps one-third to one-half of those capable pupils who drop out of school on high school graduation would continue their education with adequate financial support, the addition of scholarships may not bring the returns that one might expect. The additional scholarships may be used by those who would go to college anyway. In any case, the addition of 100,000 scholarships probably would not result in 100,000 *more* college students. However, it is not to be implied that financial aid is not needed by many pupils and the need will increase as the cost of collegiate training continues its upward climb.

What about the 100,000 or more students who do not go to college just because they do not see the need for a college education? They have the financial resources and the talent, but they do not have the desire. These students have received instruction from our mathematics teachers for twelve years; they have been counseled by our guidance specialists; they have been successful in educational tasks (top one-third of their class); they have concluded that to them and to society, more education is not of most worth. Are they correct? Who is responsible for their conclusion? Is it their teachers? Dr. Cole states, "At the present time, one of the

[1] Charles C. Cole, Jr., *Encouraging Scientific Talent* (New York: College Entrance Examination Board, 1956).

most serious crises the nation has ever faced is the shortage, almost the disappearance, of the competent, well-trained, stimulating high school science teacher. It is a breed that faces extinction." Shall we assume that Dr. Cole is permitting his educational zeal to distort a scientific attitude? From research it would be most difficult to prove or disprove the statement. Since I am a teacher, I prefer to assume that the teachers are doing a better job of teaching mathematics and science than they have ever done.

In fairness to Dr. Cole, it should be stated that he does cite a study which shows the superiority of a present-day teacher for science motivation. In the study, 19 per cent of the scientists educated since 1934 indicated the high school teacher as the prime stimulator to their science careers. Before that date the percentage was 13.

Perhaps teachers are doing a better job of teaching than they ever have even in overcrowded classrooms on inadequate salaries. Does this mean they should not and are not trying to improve their instruction? Only a casual survey of the in-service training programs and summer workshops and conferences brings evidence of the interest of the teachers. The arithmetic teachers in one city decided to invite the arithmetic teachers from neighboring schools for an all-day study of better ways to teach arithmetic. The meeting was held on the teachers' time (Saturday) and at the teachers' expense. The determination of teachers to improve their instruction is evidenced by the attendance of more than 1,500 teachers. One large industrial company gave sixty fellowships for summer seminars and workshops on science and mathematics instruction. Again the interest of the teachers was shown by more than two thousand applications. Perhaps improved instruction will help more pupils see the importance of education and the place of mathematics in modern society.

In what other ways can the noncollege-bound student be guided toward college? Can the mathematics teachers furnish more career opportunity guidance material to pupils? Some teachers do keep a display of guidance material in a reading corner or on a bulletin board. Teachers occasionally bring in scientists to talk to pupils on opportunities in science. Greater emphasis on such activities might result in more of the talented going to college.

Would a change in the content attract more youth?

Many leaders in mathematics education have suggested that much deadwood should be removed from our present mathematics curriculum and replaced with more modern ideas of mathematics. A few small groups of mathematics teachers have been attempting to improve the mathematics curriculum in light of advances in mathematics. An example of such a pioneering group is the Curriculum Committee of Illinois. Should not such experimentation be taking place in many schools and co-ordinated on a national level? The National Council of Teachers of Mathematics has recently appointed a Curriculum Committee. Will they exert leadership in this area, or is that too much to hope for?

Of course, the teachers alone cannot motivate all able pupils to pursue college courses even if it were desirable. As Dr. Cole points out, "many more [pupils] are affected by deterents outside the school." The community and the home likewise mold the aspirations of youth. The teacher as a member of the community has a responsibility and opportunity to impart information on the loss of superior pupils from our educational institutions and ways the community can encourage youth to continue their education.

One of the most recent and valuable studies of our able students who are lost to college and of ways of attracting them to college is *Encouraging Scientific Talent*, by Charles C. Cole, Jr. This book should be available to every high school teacher and guidance counselor.

• TESTING TIME

Edited by Robert S. Fouch, Florida State University, and Robert Kalin, Florida State University, Tallahassee, Florida

Answering the Russians

In the very first article of this department, as published in the January 1957 issue, your editors suggested two principles about tests:

1. A test is good (or bad) to the extent that it measures (or fails to measure) the attainment of the objectives of instruction set down by the teacher.

2. As a corollary to this, since we believe that understanding is important, we propose that student understanding of mathematical principles and student ability to apply these understandings should be measured as well as simpler skills.

In the second column, published in the February issue, we gave fifteen problems from the 1950–52 examinations for entrance to the Mechanics-Mathematics Department of the University of Moscow.

In this, the third column of Testing Time, your editors present their solutions and analyses for seven of these Russian examination questions. Only seven will be considered for reasons of time and space, and in the hope that some of our readers will feel motivated to make comments on the remaining eight for publication in this space.

Example 1(a)

Determine the exponent n (n is a positive whole number) in the expansion of

$$(\tfrac{1}{5}x+\tfrac{2}{5})^n$$

by diminishing powers of x, if the tenth member of this expansion (counting from the beginning) has the largest coefficient.

Solution:

(1) Factoring, we obtain

$$(\tfrac{1}{5})^n(x+2)^n.$$

(2) If we assume that the problem refers to a *unique* largest coefficient in the expansion, then,

$$C_8^n 2^8 < C_9^n 2^9 > C_{10}^n 2^{10}.$$

(3) The first inequality produces

$$\frac{n!}{(n-8)!8!}\cdot 2^8 < \frac{n!}{(n-9)!9!}\cdot 2^9.$$

(4) Solving, we obtain $n > 12\frac{1}{2}$.

(5) The second inequality produces

$$\frac{n!}{(n-9)!9!}\cdot 2^9 > \frac{n!}{(n-10)!10!}\cdot 2^{10}.$$

(6) Solving, we obtain $n < 14$.

(7) Therefore $n = 13$.

Comments:

1. This item tests considerable student knowledge about such topics as the binomial expansion, inequalities, and factorials. Much more power is called for than in the usual type of problem: e.g., "What is the tenth term in the expansion of $(x+2)^{13}$?"

2. Could our high school graduates who have taken four years of mathematics solve this problem? Our present programs generally pay little attention to the topic of inequalities. It is interesting to note that some curriculum groups will probably advocate more work with inequalities than is presently covered.

Example 1(b)

Solve the system of equations:

$$7\sqrt[4]{xy}-3\sqrt{xy}=4$$
$$x+y=20$$

Solution:

(1) Let $z=\sqrt[4]{xy}$; then $z^2=\sqrt[2]{xy}$ and $z^3=\sqrt{xy}$.

(2) Substituting in the first equation, we obtain $3z^3-7z^2+4=0$, whose solutions are $-\frac{2}{3}$, 2, and 1.

(3) Since $\sqrt[4]{xy}$ is assumed to be positive, we reject $-\frac{2}{3}$ and consider $\sqrt[4]{xy}=2$ and $\sqrt[4]{xy}=1$.

(4) In the first instance we obtain the system, $xy=64$ and $x+y=20$, solutions for which are $(x, y)=(4, 16)$ or $(16, 4)$.

(5) In the second instance we obtain the system, $xy=1$ and $x+y=20$, solutions for which are $(x, y)=(10-\sqrt{99}, 10+\sqrt{99})$ or $(10+\sqrt{99}, 10-\sqrt{99})$.

Comments:

1. There has been a great deal of interesting discussion about "pattern" in mathematics; e.g., See Chapter 3 in W. W. Sawyer's *Prelude to Mathematics*. The writer of this question was consciously or unconsciously clever. The good student might see the pattern of the coefficients 7, 3, and 4 in the first equation and quickly conclude that at least $xy=1$, thus leading quickly to two of the four solutions. Should the Russian examiner honor this possibility by rephrasing his question in the form: "Find two solutions for the system of equations . . . "? Wouldn't a sufficient amount of student skill with complicated algebraic technique be tested in this way?

2. Your editors developed a side interest in graphing this system of equations. This is elementary for the linear equation; but what is the graph of $7\sqrt[4]{xy}-3\sqrt{xy}=4$? This would seem extremely difficult to graph by any traditional method. But $7\sqrt[4]{xy}-3\sqrt{xy}=4$ if and only if $xy=1$ or $xy=64$; in other words, the graph of the original equation is the union of the two hyperbolas, $xy=1$, $xy=64$. A sketch of the graph of this system quickly reveals the four solutions obtained.

Example 1(c)

Let x_1 and x_2 be the roots of the equation:

$$x^2-ax+a-1=0.$$

where a is a real number. Find the value of a for which the magnitude of expression $x_1^2+x_2^2$ will be least.

Solution:

(1) According to the relationship between roots and coefficients,

$$x_1+x_2=a \quad \text{and} \quad x_1x_2=a-1$$

(2) but

$$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=a^2-2a+2.$$

(3) Using the calculus to find the minimum,

$$f(a)=a^2-2a+2$$
$$f'(a)=2a-2$$
$$a=1.$$

Comments:

1. There is no information available to your editors which indicates that the calculus is a regular part of the high school mathematics program in Russia. If not, then the Russian examinee would have to call upon another technique. Graphing the parabola and thereby determining the minimum seems the most obvious technique.

2. This item seems to have merit in the sense that the algebraic techniques demanded are not too complicated, knowledge of the relationship of roots to coefficients is required, and an understanding of the graphs of parabolas is needed.

Example 2(a)

Two trains proceeding in opposite direction, one an ordinary train going from A

to B and the other an express train from B to A, were delayed at an intermediate station C for two hours. By increasing the speed of their trains by 25 per cent, the engineers brought the trains, respectively, to B, 48 minutes late; and to A, 1 hour 36 minutes late.

Determine the initial speed of the trains if the speed of the express train was 20 kilometers per hour greater than the speed of the ordinary train, and the distance from A to B is 360 kilometers.

Solution:

(1) Letting d = the distance AC and r = the rate of the "ordinary" train, then $360-d$ = the distance CB and $r+20$ = the rate for the express train, etc.

(2) From this, and the statements as regards time given in the second sentence of the problem, we obtain:

$$\begin{cases} (1) & \dfrac{d}{r}+\dfrac{360-d}{\frac{4}{5}r}+2=\dfrac{360}{r}+\dfrac{4}{5} \\ (2) & \dfrac{360-d}{(r+20)}+\dfrac{d}{\frac{4}{5}(r+20)}+2=\dfrac{360}{r+20}+1\frac{4}{5}. \end{cases}$$

(Both of these equations equate the time actually taken on this particular trip with the scheduled time plus the delay. Equivalent but less complicated equations could be used by considering only the second "half" of the trip and the stop at C.)

(3) Solving for r and $(r+20)$ we obtain 40 and 60, respectively.

Comments:

1. Whatever word problems test, this one probably does the job fairly well.

2. The solution requires somewhat tiresome but elementary arithmetic and algebra; this portion of the student's work could have been made simpler by more appropriate selection of times and/or speeds by the examiner. Looking at this in another way, why should the examiner ask for a solution for the two rates? It would have been more economical use of testing time to have merely asked the student for the equations which would produce the correct solutions, since this is the key ability asked of a student in such a problem.

Example 2(b)

Solve the system of equations:

$$\begin{cases} \dfrac{1}{x}-\dfrac{1}{y}=\dfrac{1}{36} \\ xy^2-x^2y=324. \end{cases}$$

Solution:

(1) Transforming both equations:

$$\begin{cases} \dfrac{y-x}{xy}=\dfrac{1}{36} \\ xy(y-x)=324. \end{cases}$$

(2) These two equations indicate that the product of $y-x$ and xy must be 324, and their quotient (in that order) must be 1/36. The only suitable possibilities are the following:

$$\begin{cases} xy=108 \\ y-x=3 \end{cases} \qquad \begin{cases} xy=-108 \\ y-x=-3. \end{cases}$$

(3) Solving these two systems, we obtain the following four values for (x, y):

$$(-12, -9) \qquad (9, 12)$$

$$\left(\frac{3+3\sqrt{47}\,i}{2}, \frac{-3+3\sqrt{47}\,i}{2}\right)$$

$$\left(\frac{3-3\sqrt{47}\,i}{2}, \frac{-3-3\sqrt{47}\,i}{2}\right).$$

Comments:

1. Once again, the Russian examiner presented a system with an interesting pattern that could give the observant student a chance to display his talent. Would it have been wiser to have presented the system as given in step (1) above?

2. As several times before, we have a problem requiring tedious and complicated algebraic technique. Is this necessary? If so, shouldn't at least some of the testing time have been saved by asking for just one solution?

Example 2(c)

Determine three numbers forming a geometric progression if their sum is equal to 21 and the sum of their reciprocals is equal to 7/12.

Solution:

(1) The three required numbers are of the form x/r, x, xr, and the following equations hold:

$$\begin{cases} \dfrac{x}{r}+x+xr=21 \\ \dfrac{r}{x}+\dfrac{1}{x}+\dfrac{1}{xr}=\dfrac{7}{12}. \end{cases}$$

(2) Factoring, we obtain

$$\begin{cases} x\left(\dfrac{1}{r}+1+r\right)=21 \\ \dfrac{1}{x}\left(r+1+\dfrac{1}{r}\right)=\dfrac{7}{12}. \end{cases}$$

(3) Dividing, we find that $x^2=36$ and that $x=\pm 6$.

(4) Substituting in the first equation and simplifying leads to the two quadratics

$$\begin{cases} 2r^2-5r+2=0 \\ 2r^2+9r+2=0. \end{cases}$$

(5) The first produces the triple of numbers: 3, 6, 12; and the second produces the triple:

$$\frac{3}{2}(9+\sqrt{65}),\quad -6,\quad \frac{3}{2}(9-\sqrt{65}).$$

Comments:

And again, to obtain all solutions requires a great deal of needless work from the testing point of view. It would seem advisable to ask for three *integers* rather than three *numbers*, thus cutting the examinee's work in half.

Example 2(d)

Solve the equation:

$$\lg\sqrt{1+x}+3\lg\sqrt{1-x}=\lg\sqrt{1-x^2}+2.$$

Solution:

(1) Using the laws for logarithms,

$$\lg\sqrt{\frac{(1+x)(1-x)^3}{(1+x)(1-x)}}=2.$$

(2) Then,

$$\lg|1-x|=2$$

$$|1-x|=b^2, \text{ where } b \text{ is the base}$$

$$x=1-b^2 \quad \text{or} \quad x=1+b^2.$$

Comment:

The base has been symbolized by "b" for several reasons. First, it is not known what base was intended. Secondly, the usual bases, e and 10, lead us to the complex domain.

To avoid the complex domain, we reject $x=1+b^2$ and, when $x=1-b^2$, demand that b^2 be less than 2, a statement derived from the requirement that $1+x>0$ and $1-x>0$.

GENERAL COMMENTS

1. In 1953, applicants to the Mechanics-Mathematics Department of the University of Moscow were given three algebra problems like the seven above and three geometry problems of similar difficulty. How well did they fare? In a pamphlet whose purpose was to inform and arouse the interest of potential applicants to the University, the eminent Russian mathematician, Kolmogorov, states that very few students solved all six problems given in 1953, that four out of six solved correctly was considered a very good result, and that only those solving fewer than two were denied admission. (It is interesting to note that this statement is made in the context of trying to persuade students to overcome their fear of these examinations.)

2. Our first principle about tests requires us to answer the question, "Is the test good or bad?" by considering the objectives of mathematical instruction in Russian high schools and at the Univer-

sity of Moscow. Some clues about these objectives are supplied by Kolmogorov: " . . . in order to be accepted to a university the first requirement is to have a firm knowledge of the course given in the school and the ability on the basis of this knowledge accurately and confidently to solve the more usual, . . . standard problems."

Reading between the lines of this statement and using other information known to us, we are led to believe that there is great emphasis upon algebraic technique in Russian high school mathematics, upon skill in topics and tricks that in America are found in the more traditional college algebra courses. Thus the objectives of Russian instruction seem to be the coverage of a large percentage of material such as is found in texts like those by Hall and Knight, and by Wentworth. If this inference is correct, then we assign this test a grade of "good" in that it appears to measure such objectives.

3. There is a great deal of talk revolving around the idea of competition between the training of Russian and American students. Attempting such problems as those considered in this article might lead the reader to wonder if American students develop sufficient mathematical knowledge. Two comments seem in order. First, we must recognize that examinations are given in this country, e.g., the Stanford Competitive Examination in Mathematics, which undoubtedly require an even higher level of mathematical attainment than are demanded by these Russian questions.

4. The second comment is related to our second principle, briefly, that understanding is important. Here we think the Russian questions fail to a considerable extent. We doubt that any of these questions test understanding in a direct way. Imbedded in some items are steps which necessitate understanding of basic algebraic principles, but these are lost in a maze of manipulation.

In the words of Kolmogorov:

> No special ingenuity is required to solve the examination problems. In the majority of cases these problems are solved by progressive application of the rules and methods studied in school. Such independent thinking that is required for the solution amounts to a systematical analysis of the question following a most natural sequence. . . . At times, aiming to test by one problem the knowledge of a whole series of formulas, rules, and theorems included in the school course, the examiners structure complex conditions, giving to the problems a rather artificial and confusing form. . . . In their substance, . . . they (the problems) are considerably more elementary than the problems which are more briefly and elegantly formulated.

For these reasons, we think the questions fail to meet our second principle.

5. One could linger on a discussion of these interesting questions forever. Let us dispose of these discussions by asking two questions: Would it be possible to rephrase these questions to emphasize the measurement of mathematical understanding? Could other questions be asked about some aspects of these problems so as to measure understanding?

Annual Business Meeting

Notice is hereby given, as required by the Bylaws, that the Annual Business Meeting of the National Council of Teachers of Mathematics will be held during the Thirty-Fifth Annual Meeting, March 28–30, 1957, at the Bellevue-Stratford Hotel, Philadelphia, Pennsylvania.

• TIPS FOR BEGINNERS

Edited by Francis G. Lankford, Jr., Longwood College, Farmville, Virginia, and Joseph N. Payne, University of Wisconsin, Madison, Wisconsin

Extending number concepts

by Charles Brumfiel, Ball State Teachers College, Muncie, Indiana

When a child enrolls in the first grade, he may be able to count in a number system which consists of a few of the integers 1, 2, 3, · · ·. As he moves through the arithmetic curriculum of the grades, and as he studies algebra, significant extensions of his number concepts occur.

Probably within the first year he will realize that the counting numbers form an *infinite set*. He soon associates *zero* with the *empty set*.

After a long period of indoctrination, he accepts (albeit perhaps uneasily) calculations with fractions. Unfortunately there seems to be a tendency to present fractions in such a way that symbols like 3.14, 5%, $2\frac{1}{2}$ are generally thought *not* to represent fractions.

In high school algebra, there occur three important extensions of the number system studied in arithmetic. Students are introduced to *negative numbers*, *irrational numbers*, and *complex numbers*.

These three extensions in algebra are carried out under conditions vastly different from those which prevailed in arithmetic when fractions were added to the study of whole numbers. Our civilization would not permit us to conceal a knowledge of fractions from our students. Nearly everyone would be automatically forced by his environment to gain some degree of mastery of the rational number system even if all references to fractions were stricken from the textbooks. But it is not so with the other extensions of the number system. For example, irrational numbers are not ordinarily encountered at home, at the store, or in church. The teacher usually has it within his power to decree the very day and hour upon which his student shall encounter the concept of irrational numbers.

Since we have such control, it would seem reasonable to give considerable thought to the techniques to be used in making these extensions. We might well ask ourselves why we introduce such things as negative numbers. It is *not* merely for the purpose of enabling our pupils to read thermometers or to distinguish between eastward and westward motion. *One of the primary objectives is to create a more powerful number system to serve as a coefficient field in our mathematical investigations.* To help your students appreciate this fact I suggest you have them set up some equations which lead to the solutions of various problems. Ask them to try to work with these equations subject to the restriction that every symbol used must represent a positive number. They will soon see how limiting this restriction is. We know, of course, that analytic geometry, trigonometry, and calculus are barren without negative numbers.

I propose that a suitable introduction to negative numbers might easily be built around the following two mathematical applications.

1. A co-ordinate system on a line.
2. Exponents with base 10.

By (1) I mean to imply only that a co-ordinate system is to be described, as below.

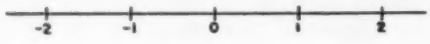


Figure 1

No reference need be made to *vectors* or *directed numbers.*

The second example can be used to give an intuitive understanding of calculations with these new numbers. The students know, or can be taught that

$$10^3=1000; \quad 10^2=100; \quad 10^1=10.$$

It should not be difficult to present the definitions

$$10^0=1;$$

$$10^{-1}=\frac{1}{10}; \quad 10^{-2}=\frac{1}{100}; \quad 10^{-3}=\frac{1}{1000}.$$

With these definitions we have at once $10^2\times10^1=10^3$; $10^{-2}\times10^{-1}=10^{-3}$; $10^3\times10^{-1}=10^2$. The additions $(-2)+(-1)=-3$ and $3+(-1)=2$ may be justified from these examples. With a little cleverness it may even be possible to introduce multiplication, since $(10^{-2})^2=10^{-4}$. Perhaps the mysterious rule $(-1)\cdot(-1)=1$ could even be presented in this same manner.

But there is little need to search mathematics for simple applications which employ negative numbers. Early in the algebra course students may start using them in the solution of simple equations. However, a skillful teacher should be able to carry out a sequence of informal proofs suggesting the validity of the usual rules for computing with negative numbers. We indicate below how these proofs may be effected for certain numerical cases.

We might begin by giving to the symbol -5 the name *the negative of five* and *defining* it by the equation

$$(-5)+5=0.$$

The negatives of other numbers are defined by analogy. Now we *assume* that the commutative, associative, and distributive laws still hold as we calculate with these new numbers. The following theorems and proofs illustrate what can be done.

Theorem 1. $6+(-4)=2$
Proof.

$$6+(-4)=2+[4+(-4)]=2+0=2$$

Theorem 2. $(-3)+(-5)=-8$
Proof.

$$8+[(-3)+(-5)]=3+5+(-3)+(-5)$$
$$=[3+(-3)]+[5+(-5)]=0$$

Now *since* $[(-3)+(-5)]$ *gives zero when we add it to* 8 *it is by definition the negative of* 8.

Theorem 3. $5+(-8)=-3$
Proof. By Theorem 2,

$$(-8)=(-5)\pm(-3).$$
$$[5+(-5)]+(-3)=0+(-3)=(-3)$$

Proof of the multiplication rules requires employment of the distributuve law.

Theorem 4. $4\times(-3)=-12$
Proof.

$$4\times[3+(-3)]=0=12+4(-3)$$

Adding (-12) to each side of this last equation we get $(-12)=4(-3)$.

Theorem 5. $(-4\times(-3)=12.$
Proof.

$$(-4)\times[3+(-3)]=(-12)$$
$$+(-4)(-3)=0$$

Adding 12 to each side of the final equation we get $(-4)(-3)=12$.

Somewhere along the way the teacher should ask, "What might we mean by *the negative of the negative of* 5?" If the definition is suggested: *the number which added to the negative of five yields zero,* then we can write

$$-(-5)=5.$$

Notice that we have used the language *the negative of* 5 rather than *negative* 5. Most algebra students do not understand the twofold usage of the word *negative* in algebra. *Each of the numbers* 5 *and* -5 *is the negative of the other.* A symobl "$-a$" may or may not represent a "negative" number in the second sense of the word negative. If $a=-3$, then $-a$ is a *positive* number. It should be understood that *negative* is a *property* which an *individual number* may or may not have. However, *the negative of* is a *relation* which either holds or does not hold between a given *pair of numbers.*

There is indeed a way to relate operations with negative numbers to *vectors.* We establish a co-ordinate system as before and associate each point with the directed line segment from zero to the point. We call these directed line segments *vectors.* Now our numbers are merely *names* for these vectors. We define addition and subtraction of these vectors in the usual geometric fashion and get rules for addition and subtraction.

We define a product like $2\times(-3)$ to be a *stretching* or doubling of the length.

We define $(-1)\times 4$ as a *rotation* of the vector 4, about the point zero through 180°. Hence $(-1)\times 4=-4$.

Now we have $(-1)\times(3)=-3$, hence we define $(-3)\times(5)=(-1)\times[3\times 5]=-15$. That is, multiplication by (-3) is a stretching followed by a rotation.

No teacher should present this illustration of the use of numbers unless he understands that he is teaching some isolated facts relating to calculations with complex numbers. In advanced algebra, students will learn to visualize addition and multiplication of complex numbers as vector addition and multiplication, and it may then be seen that the above procedures fit into a more general scheme. At that time, they will see that when the complex number $a+bi$ is multiplied by i, the vector representing the number $a+bi$ is simply rotated 90°. The multiplication by -1 is then seen as equivalent to two successive multiplications by i, or as a rotation of 180°.

When we introduce our students to new ideas, ideas which are eventually to be fitted into a great edifice of mathematical thought, it behooves us to present these new concepts in such a manner that at the dawn of deeper understanding our students will say: "Now at last the picture has fallen into place. Now I know why my teacher presented these ideas to me as he did."

Have you read?

LEAR, JOHN. "The Electron and the New 'Number' Illiteracy," *Saturday Review of Literature,* November 3, 1956, pp. 45–48.

No doubt all your students listened to the election returns and the prediction by the computers. What are the implications of this for the mathematics of all of us? It would appear we must learn the language of the robots. They predict weather, assembly production, blood pressure, plant growth, and a host of other things. Their language is symbolic; they calculate chances; they explain logical choices, and the symbols are mathematical. The concepts of computation of twenty years ago cannot successfully be applied today. Business students of today often study calculus to understand economic predictions of Univac. Who knows, will the blackboard of tomorrow be the electronic computor? I.B.M. has recently made its 704 available to all schools in the Boston area. This is a thought-provoking article. Read it.—PHILIP PEAK, *Indiana University, Bloomington, Indiana.*

National Science Foundation Institutes for teachers of science and mathematics

Opportunities for teachers to obtain additional training in science and mathematics will be considerably more numerous during the summer of 1957 and the academic year 1957–58. Dr. Alan T. Waterman, Director of the National Science Foundation, has recently announced the awarding of ninety-five grants for summer institutes and sixteen grants for academic year institutes in colleges and universities throughout the United States and its territories. These institutes will provide programs specially designed for teachers. Emphasis will be on subject matter with a view to increasing the competence of teachers as teachers, not as research workers. In addition to providing tuition and fees, the grants will provide further financial assistance to approximately 4600 teachers during the summer and 750 teachers during the academic year.

The grants for summer institutes will provide for each stipend recipient $75 per week plus $15 per week for each dependent (to a maximum of four dependents) and a travel allowance equal to one round-trip from home to institute at the rate of four cents per mile (maximum of $80). There will be no book allowance.

The Foundation grants for academic-year institutes will provide each stipend recipient with $3000 plus an allowance of $300 for each dependent (to a maximum of four dependents) and travel (two round trips from home to institute at the rate of four cents per mile, with a maximum of $80 each trip). An allowance will also be made for books.

The institute programs sponsored by the National Science Foundation have grown rapidly since they were begun with two institutes for college teachers in the summer of 1953. There were four NSF summer institutes in 1954, eleven in 1955, and twenty-five in 1956. The academic-year institutes were begun during the present academic year and two are now in progress. All academic-year institutes and eighty-six of the 1957 summer institutes will be for high school teachers. Four of the summer institutes will be for both high school and college teachers, and five will be scheduled and planned for college teachers only.

Programs for teachers of mathematics continue to be a prominent part of the institutes programs. For the coming summer there are to be thirteen institutes in mathematics for high school teachers, one for both high school and college teachers, and one for college teachers only. In addition to these, instruction in mathematics will be provided in thirty-three of the fifty institutes which offer work in more than one field. Two of the academic-year institutes are institutes in mathematics. All other academic-year institutes provide work in mathematics and several other sciences.

The locations of the 1957 summer institutes in mathematics are given below. The person to whom inquiries should be addressed is given in parentheses for each institute.

SUMMER INSTITUTES FOR HIGH SCHOOL MATHEMATICS TEACHERS ONLY

Columbia University Teachers College, New York 27, New York (H. F. Fehr)

Indiana University, Bloomington, Indiana (Mrs. M. S. Wilcox)

Miami University, Oxford, Ohio (T. C. Holyoke)

Montana State College, Bozeman, Montana (J. W. Hurst)

Polytechnic Institute of Puerto Rico, San German, Puerto Rico (Mariano García)

State Teachers College (for Junior High School Teachers), Oneonta, New York (Vera Sanford)

State University of Iowa, Iowa City, Iowa (L. A. Knowler)

University of Buffalo, Buffalo, New York (Harriet F. Montague)

University of Chicago, Chicago, Illinois (A. L. Putnam)

University of Colorado, Boulder, Colorado (B. W. Jones)

University of Massachusetts, Amherst, Massachusetts (R. W. Wagner)

University of Notre Dame, Notre Dame, Indiana (A. E. Ross)

University of Wyoming, Laramie, Wyoming (W. N. Smith)

SUMMER INSTITUTES FOR HIGH SCHOOL TEACHERS AND COLLEGE TEACHERS

University of Kansas, Lawrence, Kansas (G. B. Price)

SUMMER INSTITUTES FOR COLLEGE TEACHERS

University of Colorado, Boulder, Colorado (R. E. Langer, Department of Mathematics, University of Wisconsin, Madison, Wisconsin)

In addition to the institutes for mathematics teachers only, the National Science Foundation is sponsoring institutes for teachers of mathematics and the other sciences. The location of these institutes is given below. The person to whom inquiries should be addressed is given in parentheses for each institute.

SUMMER INSTITUTES FOR HIGH SCHOOL TEACHERS ONLY

Arizona State College, Flagstaff, Arizona (Junia E. McAlister, Department of Chemistry)

Atlanta University, Atlanta, Georgia (K. A. Huggins, Department of Chemistry)

Bucknell University, Lewisburg, Pennsylvania (L. Kieft, Department of Chemistry)

Clarkson College of Technology, Potsdam, New York (F. G. Lindsey, Director of Summer Programs)

Colorado College, Colorado Springs, Colorado (L. N. Pino, Cutler Hall)

Duke University, Durham, North Carolina (W. M. Nielsen, Department of Physics)

Iowa State College, Ames, Iowa (J. A. Greenlee, Office of Dean of Science)

Louisiana State University, Baton Rouge, Louisiana (H. B. Williams, Dept. of Chemistry)

Michigan State University, East Lansing, Mich. (F. B. Dutton, Department of Chemistry)

Morgan State College, Baltimore, Maryland (T. P. Fraser, Department of Science Education)

North Carolina College, Durham, North Carolina (W. H. Robinson, Department of Physics)

Ohio University, Athens, Ohio (Gilford Crowell)

Oklahoma A. and M. College, Stillwater, Oklahoma (J. H. Zant, Department of Mathematics)

South Dakota State College, Brookings, South Dakota (K. E. Howard, Department of Chemistry)

Southern Methodist University, Dallas, Texas (J. P. Harris, Department of Biology)

University of Alabama, University, Alabama (C. K. Arey, Department of Education)

University of Alaska, College, Alaska (W. K. Keller)

University of Arkansas, Fayetteville, Arkansas (L. F. Bailey, Department of Botany)

University of California, Berkeley, California (T. N. Barrows, University Extension)

University of California, Los Angeles, California (W. G. Young, Department of Chemistry)

University of Idaho, Moscow, Idaho (W. H. Cone, Physical Science Department)

University of Maryland, College Park, Maryland (G. W. Wharton, Department of Zoology)

University of Minnesota, Duluth, Minnesota (W. R. McEwen)

University of Minnesota, Minneapolis, Minnesota (J. W. Buchta, 225 Johnston Hall)

University of Mississippi, University, Mississippi (D. R. Hutcherson, Graduate School)

University of Missouri, Columbia, Missouri (W. J. Dale, Department of Chemistry)

University of North Carolina, Chapel Hill, North Carolina (E. A. Cameron, Department of Mathematics)

University of North Dakota, Grand Forks, North Dakota (J. D. Henderson, Department of Physics)

University of Pennsylvania, Philadelphia, Pennsylvania (P. E. Jacob, 116 College Hall)

University of South Dakota, Vermillion, South Dakota (C. R. Estee, Box 57, University Exchange)

University of Texas, Austin, Texas (R. C. Anderson, Department of Chemistry)

Virginia Polytechnic Institute, Blacksburg, Virginia (T. M. Hahn, Jr., Department of Physics)

Wesleyan University, Middletown, Connecticut (R. A. Gortner, Jr., Shanklin Laboratory of Biology)

ACADEMIC-YEAR INSTITUTES 1957–58

Harvard University, Cambridge, Massachusetts (Dean Francis Keppel, Graduate School of Education)

Ohio State University, Columbus 10, Ohio (Professor John S. Richardson, College of Education)

Oklahoma Agricultural and Mechanical College, Stillwater, Oklahoma (Professor James H. Zant, Department of Mathematics)

Oregon State College, Corvallis, Oregon (Professor Stanley E. Williamson, Department of Science Education)

Pennsylvania State University, University Park, Pennsylvania (Mr. William H. Powers, Arts and Sciences Extension)

Stanford University, Stanford University, California (Professor Harold M. Bacon, Department of Mathematics)

Washington University, St. Louis, Missouri (Dean T. F. Hall)

University of Chicago, Chicago 37, Illinois (Professor E. P. Northrop, Mathematics Staff)

University of Colorado, Boulder, Colorado (Professor William E. Briggs, Department of Mathematics)

University of Illinois, Urbana, Illinois (Professor Joseph Landin, Department of Mathematics)

University of Michigan, Ann Arbor, Michigan (Professor Freeman D. Miller, Department of Astronomy)

University of North Carolina, Chapel Hill, North Carolina (Professor Edwin C. Markham, Department of Chemistry)

University of Texas, Austin 12, Texas (Professor Robbin C. Anderson, Department of Chemistry)

University of Utah, Salt Lake City, Utah (Professor T. J. Parmley, Department of Physics)

University of Virginia, Charlottesville, Virginia (Professor James W. Cole, Department of Chemistry)

University of Wisconsin, Madison 6, Wisconsin (Professor C. H. Sorum, Department of Chemistry)

Your professional dates

The information below gives the name, date, and place of meeting with the name and address of the person to whom you may write for further information. For information about other meetings, see the previous issues of THE MATHEMATICS TEACHER. Announcements for this column should be sent at least ten weeks early to the Executive Secretary, National Council of, Teachers of Mathematics, 1201 Sixteenth Street, N.W., Washington 6, D. C.

NCTM convention dates

ANNUAL MEETING

March 28–30, 1957
Bellevue-Stratford Hotel, Philadelphia, Pennsylvania
M. Albert Linton, William Penn Charter School, Philadelphia, Pennsylvania

JOINT MEETING WITH NEA AND NSTA

July 1, 1957
Philadelphia, Pennsylvania
M. H. Ahrendt, 1201 Sixteenth Street, N. W. Washington 6, D. C.

SUMMER MEETING

August 19–21, 1957
Carleton College, Northfield, Minnesota
Margaret Linster, St. Louis Park High School, Minneapolis 16, Minnesota, or Kenneth O. May, Carleton College, Northfield, Minnesota

Other professional dates

Annual Meeting of Georgia Mathematics Council

March 22, 1957
Georgia State College of Business Administration, Atlanta, Georgia
Sara Durham, Coffee County High School, Douglas, Georgia

Sectional Meetings of the Illinois Council of Teachers of Mathematics

March 23, 1957, Granite City, Illinois
March 30, 1957, Joliet, Illinois
March 30, 1957, Carbondale, Illinois
April 10, 1957, Charleston, Illinois
April 13, 1957, Kewanee, Illinois
Francis R. Brown, Illinois State Normal University, Normal, Illinois

Tenth Annual Conference on Elementary and Secondary Mathematics

April 6, 1957

Illinois State Normal University, Normal, Illinois
Douglas R. Bey, Illinois State Normal University, Normal, Illinois

Eighth Annual Conference of the Michigan Council of Teachers of Mathematics

May 3–5, 1957
St. Mary's Lake, Battle Creek, Michigan
Margaret Lingo, Pierce Junior High School, Grosse Pointe, Michigan

Annual Meeting of the Association of Mathematics Teachers of New York State

May 10–11, 1957
Hotel Syracuse, Syracuse, New York
Randolph S. Gardner, New York State College for Teachers, Albany, New York

Women's Mathematics Club of Chicago and Vicinity

May 11, 1957
Tearoom of Mandel Brothers Department Store, Chicago, Illinois
Ruth Woerner, 11715 S. 82nd Court, Palos Park, Illinois

Chicago Elementary Teachers Mathematics Club

May 13, 1957
Toffenetti's Restaurant, 65 W. Monroe Street, Chicago, Illinois
Genevieve E. Johnson, Volta School, Chicago, Illinois

Shell Merit Fellowship Program for Mathematics and Science Teachers

July 1–August 11, 1957
Cornell University, Ithaca, New York
Phillip G. Johnson, Cornell University, Ithaca, New York

Ninth Annual Institute of the Association of Teachers of Mathematics in New England

August 21–28, 1957
Dartmouth College, Hanover, New Hampshire
Alma A. Sargent, 125 N. State Street, Concord, New Hampshire

The following meetings are institutes for teachers of mathematics sponsored by the National Science Foundation. The dates given are approximate. The person named in each case is the director of the institute. Unless stated otherwise, the mailing address of each director is at the institution given above.

June 24–July 20, 1957 (for college teachers only)
University of Colorado, Boulder, Colorado
R. E. Langer, Department of Mathematics, University of Wisconsin, Madison, Wisconsin

June 10–August 3, 1957
University of Kansas, Lawrence, Kansas
G. B. Price, Department of Mathematics

July 8–August 16, 1957
Columbia University Teachers College, New York 27, New York
H. F. Fehr, Department of Mathematics

June 17–August 9, 1957
Indiana University, Bloomington, Indiana
Mrs. M. S. Wilcox, Department of Mathematics

June 17–July 26, 1957
Miami University, Oxford, Ohio
T. C. Holyoke, Department of Mathematics

June 22–August 23, 1957
Montana State College, Bozeman, Montana
J. W. Hurst, Department of Mathematics

June 1–July 27, 1957
Polytechnic Institute of Puerto Rico, San German, P. R.
Mariano Garcia, College of A & M Arts, University of Puerto Rico, Mayagyez, P. R.

July 1–August 9, 1957 (For junior high school teachers)
State Teachers College, Oneonta, New York
Vera Sanford, Department of Mathematics

June 17–July 27, 1957
State University of Iowa, Iowa City, Iowa
L. A. Knowler, Department of Mathematics

July 8–August 2, 1957
University of Buffalo, Buffalo, New York
Harriet F. Montague, Department of Mathematics

June 24–August 2, 1957
University of Chicago, Chicago, Illinois
A. L. Putnam, 411 Eckhart Hall

June 17–July 27, 1957
University of Colorado, Boulder, Colorado
B. W. Jones, Department of Mathematics

July 1–August 16, 1957
University of Massachusetts, Amherst, Massachusetts
R. W. Wagner, Department of Mathematics

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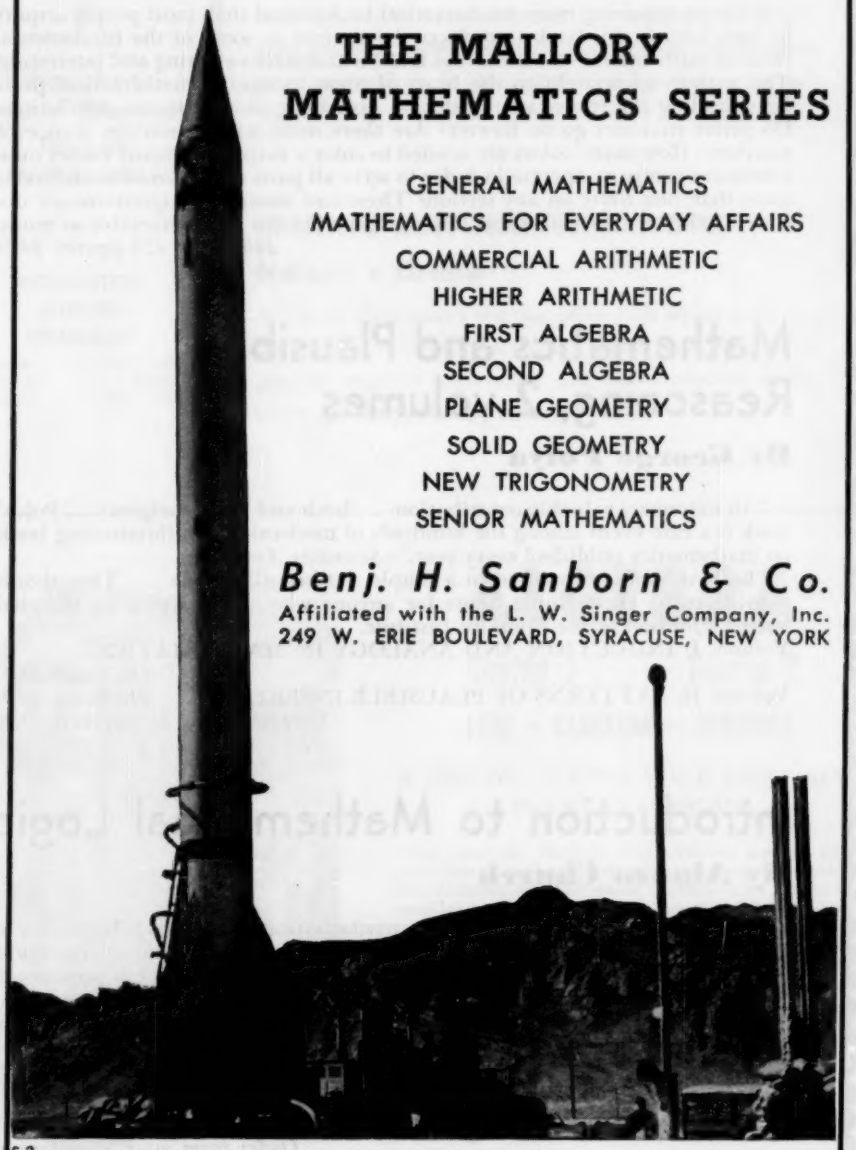

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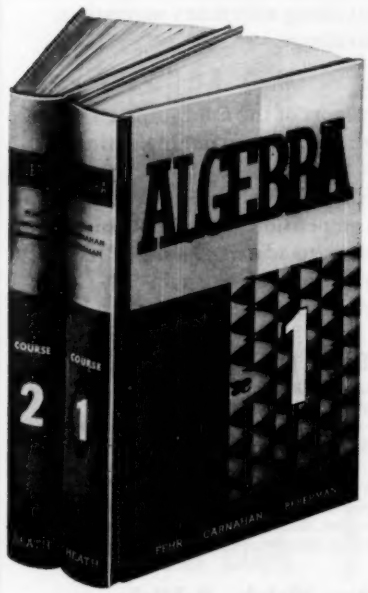

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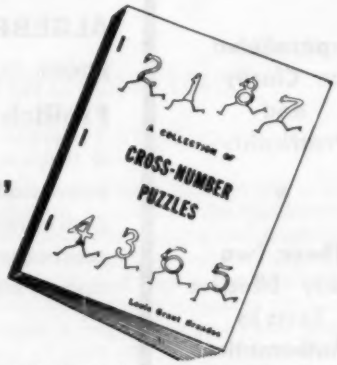

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About the Author

Louis Grant Brandes has taught mathematics in Alameda H. S. and in Encinal H. S., both in Alameda, California. He is now Vice Principal of Encinal H. S.

He has written widely in the field of recreational mathematics for such periodicals as **Mathematics Teacher, School Science and Mathematics,** and **Clearing House. His MATH. CAN BE FUN,** published by us in January, 1956, has already gone into three printings!

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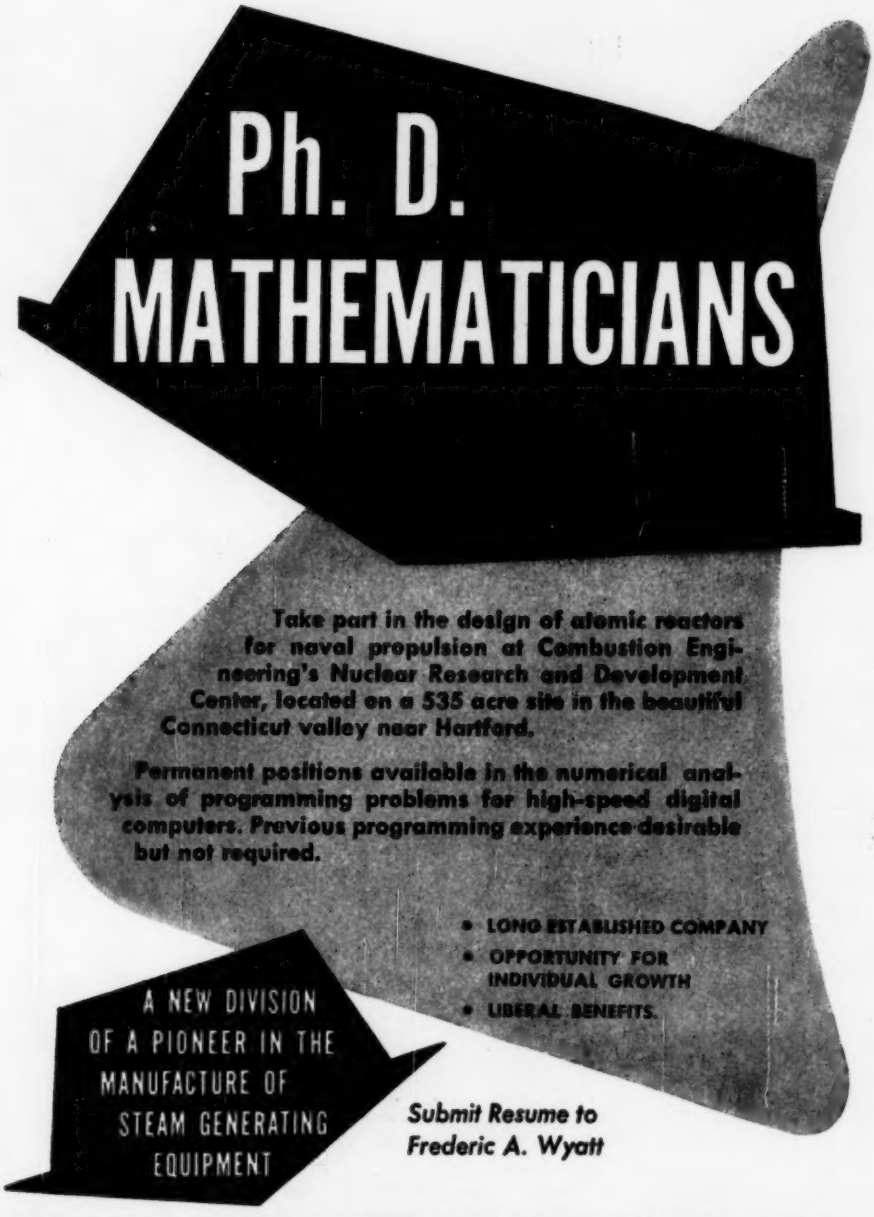
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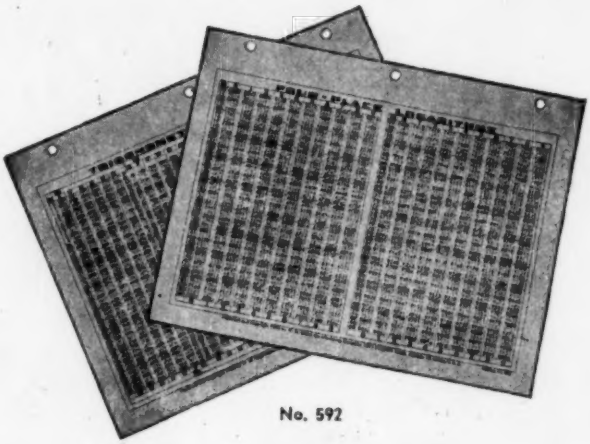

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